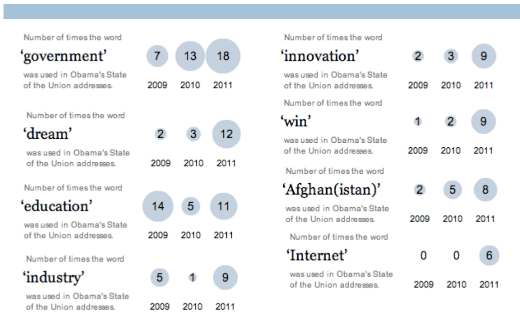
PROBABILITY

David Kauchak
CS159 – Spring 2011

Admin

- Read article for discussion on Thursday
- Assignment advice
  - test individual components of your regex first, then put them all together
  - write test cases

Corpus statistics



www.nytimes.com 1/25/2011

Basic Probability Theory: terminology

- An **experiment** has a set of potential outcomes, e.g., throw a dice, "look at" another sentence
- The **sample space** of an experiment is the set of all possible outcomes, e.g., {1, 2, 3, 4, 5, 6}
- In NLP our sample spaces tend to be ***very*** large
  - All words, bigrams, 5-grams
  - All sentences of length 20 (given a finite vocabulary)
  - All sentences
  - All parse trees over a given sentence

Basic Probability Theory: terminology

- An **event** is a subset of the sample space
- Dice rolls
  - {2}
  - {3, 6}
  - even = {2, 4, 6}
  - odd = {1, 3, 5}
- NLP
  - a particular word/part of speech occurring in a sentence
  - a particular topic discussed (politics, sports)
  - sentence with a parasitic gap
  - pick your favorite phenomena…

Events

- We're interested in probabilities of events
  - p({2})
  - p(even)
  - p(odd)
  - p(parasitic gap)
  - p(word)

Random variables

- A random variable is a mapping from the sample space to a number (think events)
- It represents all the possible values of something we want to measure in an experiment
- For example, random variable, *X*, could be the number of heads for a coin

space	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
X	3	2	2	1	2	1	1	0

- Really for notational convenience, since the event space can sometimes be irregular

Random variables

- We can then talk about the probability of the different values of a random variable
- The definition of probabilities over *all* of the possible values of a random variable defines a **probability distribution**

space	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
X	3	2	2	1	2	1	1	0

X	P(X)
3	P(X=3) = 1/8
2	P(X=2) = 3/8
1	P(X=1) = 3/8
0	P(X=0) = 1/8

Probability distribution

- To be explicit
  - A probability distribution assigns probability values to all possible values of a random variable
  - These values must be >= 0 and <= 1
  - These values must sum to 1 for all possible values of the random variable

X	P(X)
3	P(X=3) = 1/2
2	P(X=2) = 1/2
1	P(X=1) = 1/2
0	P(X=0) = 1/2

X	P(X)
3	P(X=3) = -1
2	P(X=2) = 2
1	P(X=1) = 0
0	P(X=0) = 0

Unconditional/prior probability

- Simplest form of probability is
  - P(X)
- Prior probability: without any additional information, what is the probability
  - What is the probability of a heads?
  - What is the probability of a sentence containing a pronoun?
  - What is the probability of a sentence containing the word "banana"?
  - What is the probability of a document discussing politics?
  - …

Joint distribution

- We can also talk about probability distributions over multiple variables
- P(X,Y)
  - probability of X *and* Y
  - a distribution over the cross product of possible values

NLPPass	P(NLPPass)
true	0.89
false	0.11

EngPass	P(EngPass)
true	0.92
false	0.08

NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

Joint distribution

- Still a probability distribution
  - all values between 0 and 1, inclusive
  - all values sum to 1
- *All* questions/probabilities of the two variables can be calculated from the joint distribution

NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

What is P(ENGPass)?

Joint distribution

- Still a probability distribution
  - all values between 0 and 1, inclusive
  - all values sum to 1
- *All* questions/probabilities of the two variables can be calculate from the joint distribution

NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

0.92

How did you figure that out?

Joint distribution

$$P(x) = \sum_{y \in Y} p(x,y)$$

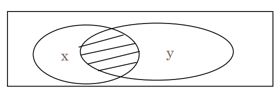
NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

Conditional probability

- As we learn more information, we can update our probability distribution
- P(X|Y) models this (read "probability of X *given* Y")
  - What is the probability of a heads *given* that both sides of the coin are heads?
  - What is the probability the document is about politics, given that it contains the word "Clinton"?
  - What is the probability of the word "banana" given that the sentence also contains the word "split"?
- Notice that it is still a distribution over the values of X

Conditional probability

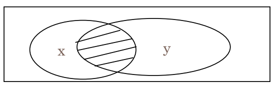
$$p(X \mid Y) = \ ?$$



In terms of pior and joint distributions, what is the conditional probability distribution?

Conditional probability

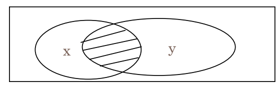
$$p(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$



Given that y has happened, in what proportion of those events does x also happen

Conditional probability

$$p(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$



Given that y has happened, what proportion of those events does x also happen

NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

What is:
p(NLPPass=true | EngPass=false)?

Conditional probability

NLPPass AND EngPass	P(NLPPass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

$$p(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$

What is:
p(NLPPass=true | EngPass=false)?

$$\frac{P(true, false) = 0.01}{P(EngPass = false) = 0.01 + 0.07 = 0.08} = 0.125$$

Notice this is very different than p(NLPPass=true) = 0.89

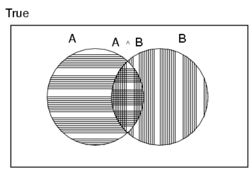
A note about notation

- When talking about a particular assignment, you should technically write p(X=x), etc.
- However, when it's clear , we'll often shorten it
- Also, we may also say P(X) or p(x) to generically mean any particular value, i.e. P(X=x)

$$\frac{P(true, false) = 0.01}{P(EngPass = false) = 0.01 + 0.07 = 0.08} = 0.125$$

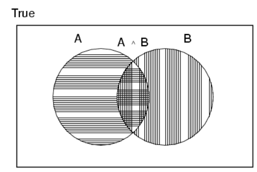
Properties of probabilities

- P(*A* or *B*) = ?



Properties of probabilities

- P(*A* or *B*) = P(*A*) + P(*B*) - P(*A*,*B*)



Properties of probabilities

- P(¬E) = 1– P(E)
- More generally:
  - Given events E = $e_1, e_2, \ldots, e_n$

$$p(e_i) = 1 - \sum_{j=1:n, j \neq i} p(e_j)$$

- P(E1, E2) ≤ P(E1)

Chain rule (aka product rule)

$$p(X \mid Y) = \frac{P(X,Y)}{P(Y)} \Rightarrow p(X,Y) = P(X \mid Y)P(Y)$$

We can view calculating the probability of X *AND* Y occurring as two steps:

1. Y occurs with some probability P(Y)
2. Then, X occurs, given that Y has occured

or you can just trust the math… ☺

Chain rule

$$p(X,Y,Z) = P(X \mid Y,Z)P(Y,Z)$$
$$p(X,Y,Z) = P(X,Y \mid Z)P(Z)$$
$$p(X,Y,Z) = P(X \mid Y,Z)P(Y \mid Z)P(Z)$$
$$p(X,Y,Z) = P(Y,Z \mid X)P(X)$$

$$p(X_1,X_2,\ldots,X_n) = \ ?$$

Applications of the chain rule

- We saw that we could calculate the individual prior probabilities using the joint distribution

$$p(x) = \sum_{y \in Y} p(x,y)$$

- What if we don't have the joint distribution, but do have conditional probability information:
  - P(Y)
  - P(X|Y)

$$p(x) = \sum_{y \in Y} p(y)p(x \mid y)$$

Bayes' rule (theorem)

$$p(X \mid Y) = \frac{P(X,Y)}{P(Y)} \Rightarrow p(X,Y) = P(X \mid Y)P(Y)$$

$$p(Y \mid X) = \frac{P(X,Y)}{P(X)} \Rightarrow p(X,Y) = P(Y \mid X)P(X)$$

$$p(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

Bayes rule

- Allows us to talk about P(Y|X) rather than P(X|Y)
- Sometimes this can be more intuitive
- Why?

$$p(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

Bayes rule

- p(disease | symptoms)
  - For everyone who had those symptoms, how many had the disease?
  - p(symptoms|disease)
    - For everyone that had the disease, how many had this symptom?
- p(linguistic phenomena | features)
  - For all examples that had those features, how many had that phenomena?
  - p(features | linguistic phenomena)
    - For all the examples with that phenomena, how many had this feature
- p(cause | effect) vs. p(effect | cause)

Gaps

V

I just won't put these away.

direct object

These, I just won't put away.

filler

I just won't put ___ away.

gap

Gaps

What did you put ___ away?

gap

The socks that I put ___ away.

gap

Gaps

Whose socks did you fold ___ and put ___ away?

gap gap

Whose socks did you fold ___ ?

gap

Whose socks did you put ___ away?

gap

Parasitic gaps

These I'll put ___ away without folding ___.
gap gap

These I'll put ___ away.
gap

These without folding ___ .
gap

Parasitic gaps

These I'll put ___ away without folding ___.
gap gap

1. Cannot exist by themselves (parasitic)

These I'll put my pants away without folding ___ .
gap

2. They're optional

These I'll put ___ away without folding them.
gap

Parasitic gaps

- http://literalminded.wordpress.com/2009/02/10/dougs-parasitic-gap/

Frequency of parasitic gaps

- Parasitic gaps occur on average in 1/100,000 sentences
- Problem:
  - Joe Linguist has developed a complicated set of regular expressions to try and identify parasitic gaps. If a sentence has a parasitic gap, it correctly identifies it 95% of the time. If it doesn't, it will incorrectly say it does with probability 0.005. Suppose we run it on a sentence and the algorithm says it is a parasitic gap, what is the probability it actually is?

Prob of parasitic gaps

Joe Linguist has developed a complicated set of regular expressions to try and identify parasitic gaps. If a sentence has a parasitic gap, it correctly identifies it 95% of the time. If it doesn't, it will incorrectly say it does with probability 0.005. Suppose we run it on a sentence and the algorithm says it is a parasitic gap, what is the probability it actually is?

G = gap
T = test positive

What question do we want to ask?

Prob of parasitic gaps

Joe Linguist has developed a complicated set of regular expressions to try and identify parasitic gaps. If a sentence has a parasitic gap, it correctly identifies it 95% of the time. If it doesn't, it will incorrectly say it does with probability 0.005. Suppose we run it on a sentence and the algorithm says it is a parasitic gap, what is the probability it actually is?

G = gap
T = test positive

$$p(g \mid t) = ?$$

Prob of parasitic gaps

Joe Linguist has developed a complicated set of regular expressions to try and identify parasitic gaps. If a sentence has a parasitic gap, it correctly identifies it 95% of the time. If it doesn't, it will incorrectly say it does with probability 0.005. Suppose we run it on a sentence and the algorithm says it is a parasitic gap, what is the probability it actually is?

G = gap
T = test positive

$$p(g \mid t) = \frac{p(t \mid g)p(g)}{p(t)}$$

$$= \frac{p(t \mid g)p(g)}{\sum_{g \in G} p(g)p(t \mid g)} = \frac{p(t \mid g)p(g)}{p(g)p(t \mid g) + p(\bar{g})p(t \mid \bar{g})}$$

Prob of parasitic gaps

Joe Linguist has developed a complicated set of regular expressions to try and identify parasitic gaps. If a sentence has a parasitic gap, it correctly identifies it 95% of the time. If it doesn't, it will incorrectly say it does with probability 0.005. Suppose we run it on a sentence and the algorithm says it is a parasitic gap, what is the probability it actually is?

G = gap
T = test positive

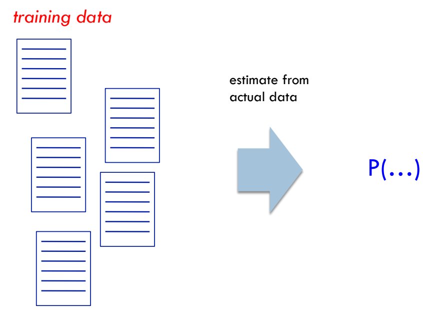
$$p(g \mid t) = \frac{p(t \mid g)p(g)}{p(g)p(t \mid g) + p(\bar{g})p(t \mid \bar{g})}$$

$$= \frac{0.95 * 0.00001}{0.00001 * 0.95 + 0.99999 * 0.005} \approx 0.002$$

Obtaining probabilities

- We've talked a lot about probabilities, but not where they come from
  - What is the probability of "the" occurring in a sentence?
  - What is the probability of "Middlebury" occurring in a sentence?
  - What is the probability of "I think today is a good day to be me" as a sentence?

Obtaining probabilities



Estimating probabilities

- What is the probability of "the" occurring in a sentence?

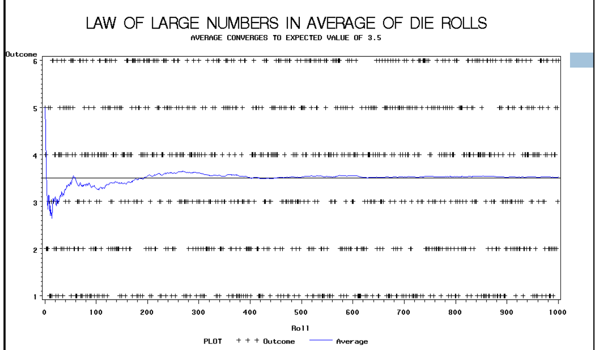
We don't know!

We can ***estimate*** that based on data, though:

$$\frac{\text{number of sentences with "the"}}{\text{total number of sentences}}$$

Maximum likelihood estimation

- Intuitive
- Sets the probabilities so as to maximize the probability of the training data
- Problems?
  - Amount of data
    - particularly problematic for rare events
  - Is our training data representative



Back to parasitic gaps

- Say the actual probability is 1/100,000
- We don't know this, though, so we're estimating it from a small data set of 10K sentences
- What is the probability that, by chance, we have a parasitic gap sentence in our sample?

Back to parasitic gaps

- p(not_parasitic) = 0.99999
- $p(\text{not\_parasitic})^{10000} \approx 0.905$ is the probability of us NOT finding one
- So, probability of us finding one is ~10%, in which case we would incorrectly assume that the probability is 1/10,000 (10 times too large)
- Remember Zipf's law from last time… NLP is all about rare events!