TF-IDF

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cs458
Fall 2012
adapted from:
http://www.stanford.edu/class/cs276/handouts/lecture6-tfidf.ppt

Administrative

- Homework 2 due Thursday
- Assignment 2 out… get started!
- Popular media article will be posted for Thursday to read and discuss
  - make sure to read it ☺

Variable byte codes

100000011000010100000100 11110001

Still seems wasteful

What is the major challenge for these variable length codes?

We need to know the length of the number!

Idea: Encode the length of the number so that we know how many bits to read

Gamma codes

Represent a gap as a pair *length* and *offset*

offset is *G* in binary, with the leading bit cut off

- 13 → 1101 → 101
- 17 → 10001 → 0001
- 50 → 110010 → 10010

length is the length of offset

- 13 (offset 101), it is 3
- 17 (offset 0001), it is 4
- 50 (offset 10010), it is 5

Encoding the length

We've stated *what* the length is, but not *how* to encode it

What is a requirement of our length encoding?

- Lengths will have variable length (e.g. 3, 4, 5 bits)
- We must be able to decode it without any ambiguity

Any ideas?

Unary code

- Encode a number *n* as *n* 1's, followed by a 0, to mark the end of it
- 5 → 111110
- 12 → 1111111111110

Gamma code examples

number	length	offset	γ-code
0			
1			
2			
3			
4			
9			
13			
24			
511			
1025			

Gamma code examples

number	length	offset	γ-code
0			none
1	0		0
2	10	0	10,0
3	10	1	10,1
4	110	00	110,00
9	1110	001	1110,001
13	1110	101	1110,101
24	11110	1000	11110,1000
511	111111110	11111111	111111110,11111111
1025	11111111110	0000000001	11111111110,0000000001

Gamma seldom used in practice

Machines have word boundaries – 8, 16, 32 bits

Compressing and manipulating at individual bit-granularity will slow down query processing

Variable byte alignment is potentially more efficient

Regardless of efficiency, variable byte is conceptually simpler at little additional space cost

RCV1 compression

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, k = 4	7.1
with blocking & front coding	5.9
collection (text, xml markup etc)	3,600.0
collection (text)	960.0
Term-doc incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, γ–encoded	101.0

TDT token normalization

normalization	terms	% change
none	120K	-
number folding	117K	3%
lowercasing	100K	17%
stemming	95K	25%
stoplist	120K	0%
number & lower & stoplist	97K	20%
all	78K	35%

What normalization technique(s) should we use?

Ranked retrieval

So far, our queries have all been Boolean
- Documents either match or don't

Good for expert users with precise understanding of their needs and the collection

Also good for applications: can easily consume 1000s of results
- Not good for the majority of users
- Most users incapable of writing Boolean queries (or they are, but they think it's too much work)

More importantly: most users don't want to wade through 1000s of results

Problem with Boolean search: feast or famine

Boolean queries often result in either too few (=0) or too many (1000s) results.

Query 1: "*standard user dlink 650*" → 200,000 hits
Query 2: "*standard user dlink 650 no card found*": 0 hits

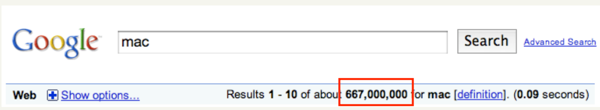
It takes skill to come up with a query that produces a manageable number of hits

With a ranked list of documents it does not matter how large the retrieved set is

Scoring as the basis of ranked retrieval

We want to return in order the documents most likely to be useful to the searcher

Assign a score that measures how well document and query "match"



Query-document matching scores

We need a way of assigning a score to a query/document pair

Why isn't it just for a score for a document?

Besides whether or not a query (or query word) occurs in a document, what other indicators might be useful?

- How many *times* the word occurs in the document
- Where the word occurs
- How "important" is the word – for example, a vs. motorcycle
- …

Recall: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrix

Consider the number of occurrences of a term in a document:

- Each document is a count vector in $\mathbb{N}^{V}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

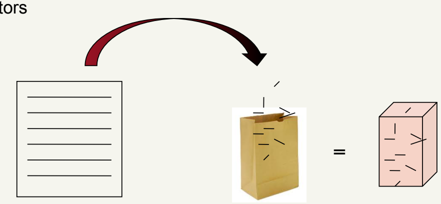
What information is lost with this representation?

Bag of words representation

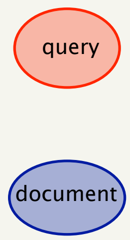
Represent a document by the occurrence counts of each word

Ordering of words is lost

John is quicker than Mary and *Mary is quicker than John* have the same vectors

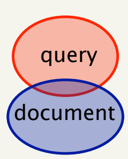


Boolean queries: another view



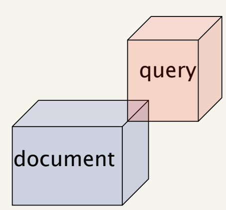
For the boolean representation, we can view a query/document as a set of words

Boolean queries: another view



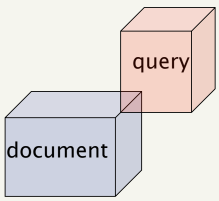
We want to return those documents where there is an overlap, i.e. intersection between the two sets

Bag of words



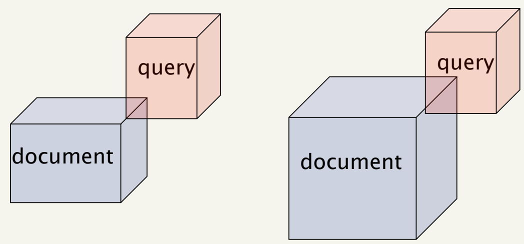
What is the notion of “intersection” for the bag or words model?

Bag of words



Want to take into account term frequency

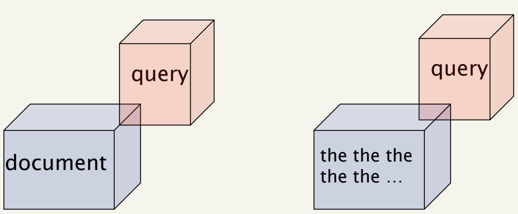
Some things to be careful of…



Say I take the document and simply append it to itself. What happens to the overlap?

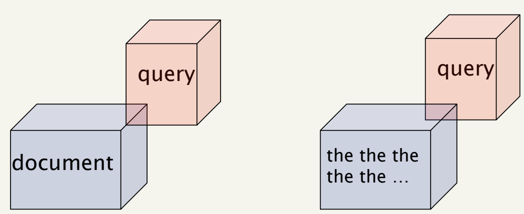
Need some notion of the length of a document

Some things to be careful of…



What about a document that contains only frequent words, e.g. the?

Some things to be careful of…



Need some notion of the importance of words

Documents as vectors

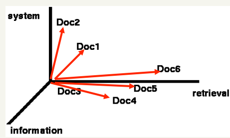
We have a |V|-dimensional vector space

Terms are axes of the space

Documents are points or vectors in this space

Very high-dimensional: hundreds of millions of dimensions when you apply this to a web search engine

This is a very sparse vector - most entries are zero

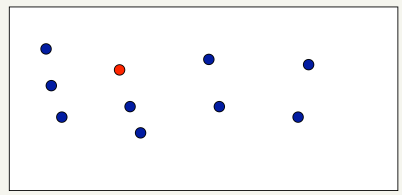


Julius Caesar
73
157
227
10
0
0
0

Queries as vectors

Key idea 1: Do the same for queries: represent them as vectors in the space

Key idea 2: Rank documents according to their proximity to the query in this space



How should we rank documents?

|V| dimensional space

Formalizing vector space proximity

We have points in a |V| dimensional space

How can we measure the proximity of documents in this space?

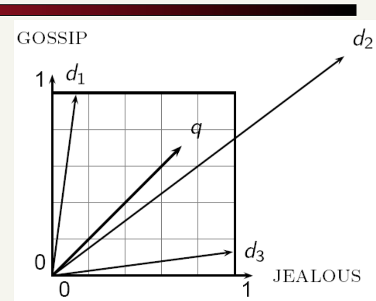
First cut: distance between two points

Euclidean distance?

Why distance is a bad idea

Which document is closer using Euclidian distance?

Which do you think should be closer?

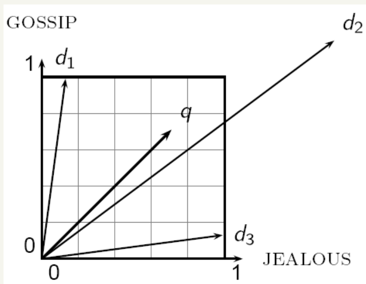


Issues with Euclidian distance

the Euclidean distance between q and d_2 is large

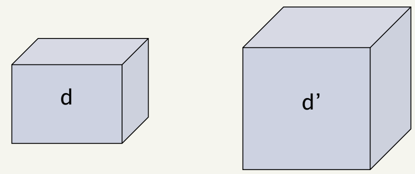
but, the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar

This is not what we want!



Use angle instead of distance

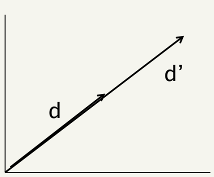
back to our thought experiment: take a document d and append it to itself. Call this document d′



“Semantically” d and d′ have the same content

Use angle instead of distance

The Euclidean distance between the two documents can be quite large



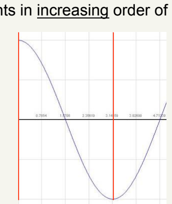
The angle between the two documents is 0, corresponding to maximal similarity

From angles to cosines

Cosine is a monotonically decreasing function for the interval [0º, 180º]

The following two notions are equivalent.

- Rank documents in <u>decreasing</u> order of the angle between query and document
- Rank documents in <u>increasing</u> order of cosine(query,document)



cosine(query,document)

How do we calculate the cosine between two vectors?

cosine(query,document)

If they are unit length:

Dot product

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

cos(*q*,*d*) is the cosine similarity of *q* and *d* … or, equivalently, the cosine of the angle between *q* and *d*.

"unit length" vectors

What is a "unit vector" or "unit length vector"?

Are our vectors unit length?

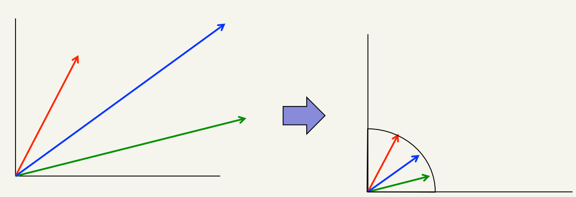
	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

No… we need some notion of the length of a document

Length normalization

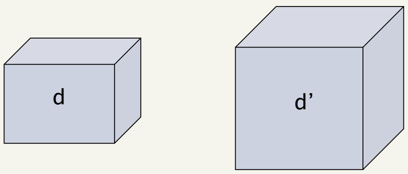
A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$



Length normalization

What effect will this have on d and d'?



they will have identical vectors after length-normalization

cosine(query,document)

Dot product   Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q} \bullet \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \bullet \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

cos(*q*,*d*) is the cosine similarity of *q* and *d* … or, equivalently, the cosine of the angle between *q* and *d*.

Cosine similarity with 3 documents

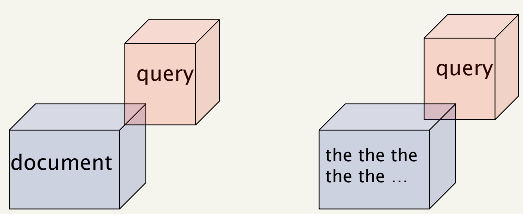
How similar are the novels:

SaS: *Sense and Sensibility*
PaP: *Pride and Prejudice*
WH: *Wuthering Heights*

Term frequencies (counts)

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6

Some things to be careful of…



Need some notion of the importance of words

Term importance

Rare terms are more informative than frequent terms

- Recall stop words

Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)

A document containing this term is very likely to be relevant to the query *arachnocentric*

We want a high weight for rare terms like *arachnocentric*

Ideas?

Document frequency

We will use document frequency (df) to capture this in the score

Terms that occur in many documents are weighted *less*, since overlapping with these terms is very likely

- In the extreme case, take a word like the that occurs in EVERY document

Terms that occur in only a few documents are weighted *more*

Collection vs. Document frequency

The collection frequency of is the number of occurrences in the collection, counting multiple occurrences

Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

Which word is a better search term (and should get a higher weight)?

Document frequency

How does “importance” or “informativeness” relate to document frequency?

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

Inverse document frequency

df_t is the <u>document</u> frequency of *t*: the number of documents that contain *t*

- df is a measure of the informativeness of *t*

We define the idf (inverse document frequency) of *t* by

$$\mathrm{idf}_t = \log \frac{N}{\mathrm{df}_t}$$

where *N* is the number of documents in the collection

what does the log do?

Inverse document frequency

$$\mathrm{idf}_t = \log \frac{N}{\mathrm{df}_t}$$

Why do we have N here?

normalizes for corpus size:
N/df_t = proportion of documents containing term *t*

idf example, suppose *N*= 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

There is one idf value for each term *t* in a collection.

idf example, suppose *N*= 1 million

term	df_t	idf_t
calpurnia	1	
animal	100	
sunday	1,000	
fly	10,000	
under	100,000	
the	1,000,000	

What if we didn't use the log?

$$\mathrm{idf}_t = \log \frac{N}{\mathrm{df}_t}$$

idf example, suppose *N*= 1 million

term	df_t	idf_t
calpurnia	1	1,000,000
animal	100	10,000
sunday	1,000	1,000
fly	10,000	100
under	100,000	10
the	1,000,000	1

The log dampens the scores

$$\mathrm{idf}_t = \log \frac{N}{\mathrm{df}_t}$$

Putting it all together

We have a notion of term frequency overlap

We have a notion of term importance

We have a similarity measure (cosine similarity)

Can we put all of these together?

Define a *weighting* for each term

The tf-idf weight of a term is the product of its tf weight and its idf weight

$$w_{t,d} = \mathrm{tf}_{t,d} \times \log N / \mathrm{df}_t$$

tf-idf weighting

$$w_{t,d} = \mathrm{tf}_{t,d} \times \log N / \mathrm{df}_t$$

Best known weighting scheme in information retrieval

Increases with the number of occurrences within a document

Increases with the rarity of the term in the collection

Works surprisingly well!

Works in many other application domains

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

We then calculate the similarity using cosine similarity with these vectors

Burstiness

Take a rare word like *arachnocentric*

What is the likelihood that *arachnocentric* occurs in a document?

Given that you've seen it once, what is the likelihood that you'll see it again?

Does this have any impact on our model?

Log-frequency weighting

Want to reduce the effect of multiple occurrences of a term

A document about "Clinton" will have "Clinton" occurring many times

Rather than use the frequency, us the log of the frequency

$$w_{t,d} = \begin{cases} 1 + \log \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.

Cosine similarity with 3 documents

How similar are the novels:

SaS: *Sense and Sensibility*
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Term frequencies (counts)

term	SaS	PaP	WH
affection	115	58	20
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gossip	2	0	6

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈
0.789 ∗ 0.832 + 0.515 ∗ 0.555 + 0.335 ∗ 0.0 + 0.0 ∗ 0.0 ≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1+\log(\mathrm{tf}_{t,d})$	t (idf)	$\log\frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5+\frac{0.5\times\mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0,\log\frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases}1 & \text{if } \mathrm{tf}_{t,d}>0\\ 0 & \text{otherwise}\end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha<1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t\in d}(\mathrm{tf}_{t,d}))}$				

Why is the base of the log in idf immaterial?

Weighting may differ in queries vs. documents

Many search engines allow for different weightings for queries vs documents

To denote the combination in use in an engine, we use the notation qqq.ddd with the acronyms from the previous table

Example: ltn.ltc means:

- Query: logarithmic tf (l in leftmost column), idf (t in second column), no normalization …
- Document logarithmic tf, no idf and cosine normalization

Is this a bad idea?

tf-idf example: ltn.lnc
(log idf none . log none cosine)

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query				Document			Prod
	tf-raw	tf-wt	df	idf	wt	tf-raw	tf-wt	n'lized	
auto	0	0	5000	2.3	0	1			
best	1	1	50000	1.3	1.3	0			
car	1	1	10000	2.0	2.0	1			
insurance	1	1	1000	3.0	3.0	2			

Doc length = $\sqrt{1^2+0^2+1^2+1^2}\approx 1.92$

tf-idf example: ltn.lnc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query					Document			Prod
	tf-raw	tf-wt	df	idf	wt	tf-raw	tf-wt	n'lized	
auto	0	0	5000	2.3	0	1	1	0.52	0
best	1	1	50000	1.3	1.3	0	0	0	0
car	1	1	10000	2.0	2.0	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	0.677	2.04

Doc length = $\sqrt{1^2+0^2+1^2+1.3^2}\approx 1.92$

Score = 0+0+1.04+2.04 = 3.08

Summary – vector space ranking

Represent the query as a weighted tf-idf vector

Represent each document as a weighted tf-idf vector

Compute the cosine similarity score for the query vector and each document vector

Rank documents with respect to the query by score

Return the top K (e.g., K = 10) to the user