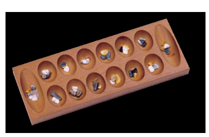
http://www.youtube.com/watch?v=LcPWEMwGJVQ

Adversarial Search

CS311
David Kauchak
Spring 2013

Some material borrowed from :
Sara Owsley Sood and others

Admin



- Reading/book?
- Assignment 2
  - On the web page
  - 3 parts
  - Anyone looking for a partner?
  - Get started!
- Written assignments
  - Make sure to look at them asap!
  - Post next written assignment soon

A quick review of search

Rational thinking via search – determine a plan of actions by searching from starting state to goal state

Uninformed search vs. informed search
- what's the difference?
- what are the techniques we've seen?
- pluses and minuses?

Heuristic design
- admissible?
- dominant?

Why should we study games?

Clear success criteria

Important historically for AI

Fun ☺

Good application of search
- hard problems (chess 35^{100} nodes in search tree, 10^{40} legal states)

Some real-world problems fit this model
- game theory (economics)
- multi-agent problems

Types of games

What are some of the games you've played?

Types of games: game properties

single-player vs. 2-player vs. multiplayer

Fully observable (perfect information) vs. partially observable

Discrete vs. continuous

real-time vs. turn-based

deterministic vs. non-deterministic (chance)

Strategic thinking $\stackrel{?}{=}$ intelligence

For reasons previously stated, two-player games have been a focus of AI since its inception…

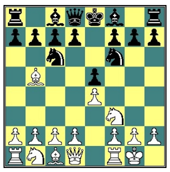
Begs the question: Is strategic thinking the same as intelligence?

Strategic thinking $\stackrel{?}{=}$ intelligence

Humans and computers have different relative strengths in these games:

humans

good at evaluating the strength of a board for a player



computers

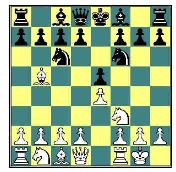
good at looking ahead in the game to find winning combinations of moves

Strategic thinking $\stackrel{?}{=}$ intelligence

How could you figure out how humans approach playing chess?

humans

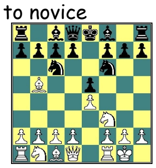
good at evaluating the strength of a board for a player



How humans play games…

An experiment (by deGroot) was performed in which chess positions were shown to novice and expert players…

- experts could reconstruct these perfectly
- novice players did far worse…



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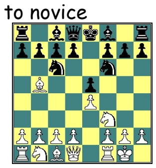
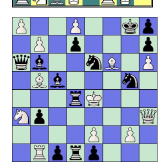
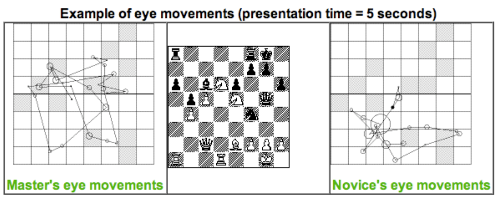


Random chess positions (not legal ones) were then shown to the two groups

- experts and novices did just as badly at reconstructing them!

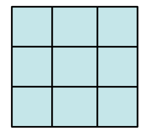


People are still working on this problem…



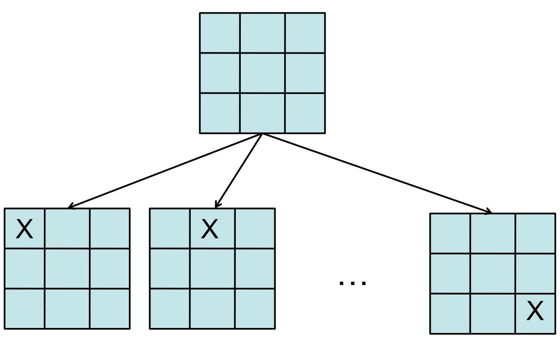
http://people.brunel.ac.uk/~hsstffg/frg-research/chess_expertise/

Tic Tac Toe as search

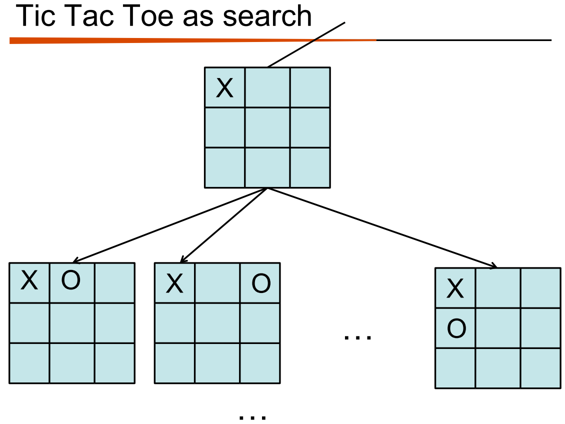


How can we pose this as a search problem?

Tic Tac Toe as search



Tic Tac Toe as search



Tic Tac Toe as search

Eventually, we'll get to a leaf

X	X	O
X	O	O
X	O	X

+1

O	X	O
X	X	O
X	O	X

0

…

X	X	O
O	X	O
X		O

-1

The **UTILITY** of a state tells us how good the states are.

Defining the problem

INITIAL STATE – board position and the player whose turn it is

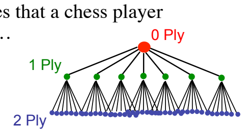
SUCCESSOR FUNCTION– returns a list of (move, next state) pairs

TERMINAL TEST – is game over? Are we in a terminal state?

UTILITY FUNCTION – (objective or payoff func) gives a numeric value for terminal states (ie – chess – win/lose/draw +1/-1/0, backgammon +192 to -192)

Games' Branching Factors

On average, there are ~35 possible moves that a chess player can make from any board configuration…



18 Ply!!

Hydra at home in the United Arab Emirates…

Branching Factor Estimates for different two-player games

Tic-tac-toe	4
Connect Four	7
Checkers	10
Othello	30
Chess	35
Go	300

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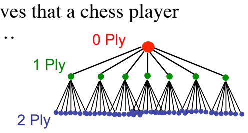

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Boundaries for qualitatively different games…

Games' Branching Factors

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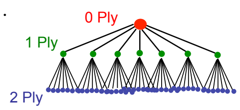


Branching Factor Estimates
for different two-player games

	Game	Branching factor
"solved" games	Tic-tac-toe	4
	Connect Four	7
CHINOOK (2007) →	Checkers	10
computer-dominated	Othello	30
	Chess	35
human-dominated	Go	300

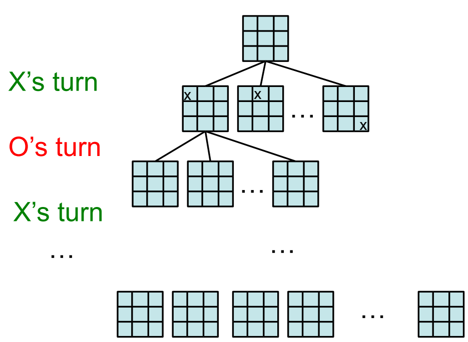
Games vs. search problems?

Opponent!
- unpredictable/uncertainty
- deal with opponent strategy

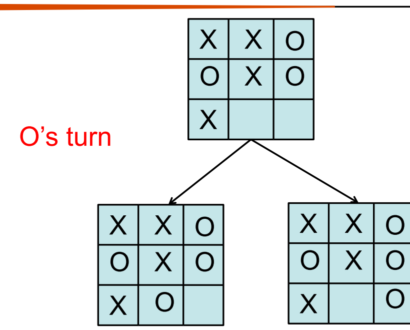
Time limitations
- must make a move in a reasonable amount of time
- can't always look to the end

Path costs
- not about moves, but about UTILITY of the resulting state/winning

Back to Tic Tac TOe



I'm X, what will 'O' do?

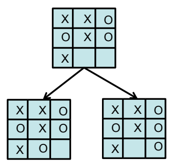


Minimizing risk

The computer doesn't know what move O (the opponent) will make

It can assume, though, that it will try and make the best move possible

Even if O actually makes a different move, we're no worse off

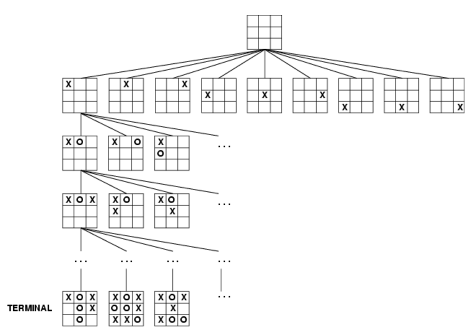


Optimal Strategy

An Optimal Strategy is one that is at least as good as any other, no matter what the opponent does

- If there's a way to force the win, it will
- Will only lose if there's no other option

How can X play optimally?

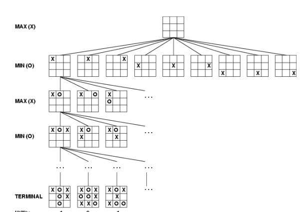


How can X play optimally?

Start from the leaves and propagate the utility up:

- if X's turn, pick the move that maximizes the utility
- if O's turn, pick the move that minimizes the utility

Is this optimal?



Minimax Algorithm: An Optimal Strategy

```
minimax(state) =
        - if state is a terminal state
                Utility(state)
        - if MAX's turn
                return the maximum of minimax(...)
                on all successors of current state
        - if MIN's turn
                return the minimum  of minimax(…)
                on all successors to current state
```

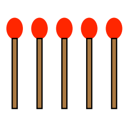
- Uses recursion to compute the "value" of each state
- Proceeds to the leaves, then the values are "backed up" through the tree as the recursion unwinds
- What type of search is this?
- What does this assume about how MIN will play? What if this isn't true?

```
def minimax(state):
  for all actions a in actions(state):
    return the a with the largest minValue(result(state,a))

def maxValue(state):
  if state is terminal:
    return utility(state)
  else:
    # return the a with the largest minValue(result(state,a))
    value = -∞
    for all actions a in actions(state):
      value = max(value, minValue(result(state,a))
    return value

def minValue(state):
  if state is terminal:
    return utility(state)
  else:
    # return the a with the smallest maxValue(result(state,a))
    value = +∞
    for all actions a in actions(state):
      value = min(value, maxValue(result(state,a))
    return value
```

ME: Assume the opponent will try and minimize value, maximize my move

OPPONENT: Assume I will try and maximize my value, minimize his/her move

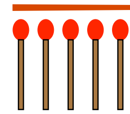
Baby Nim



Take 1 or 2 at each turn
Goal: take the last match

What move should I take?

Baby Nim



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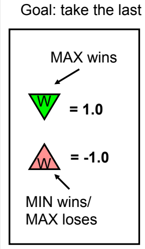
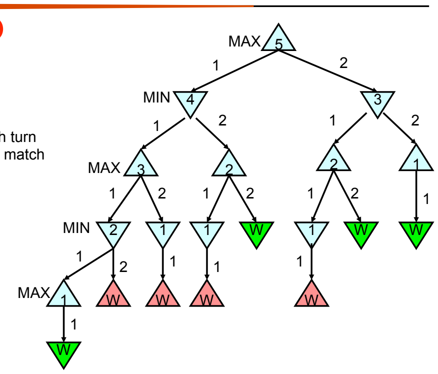
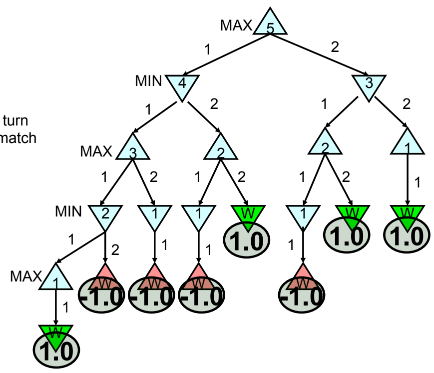




Baby Nim

Take 1 or 2 at each turn
Goal: take the last match

MAX wins
W = 1.0
W = -1.0
MIN wins/
MAX loses



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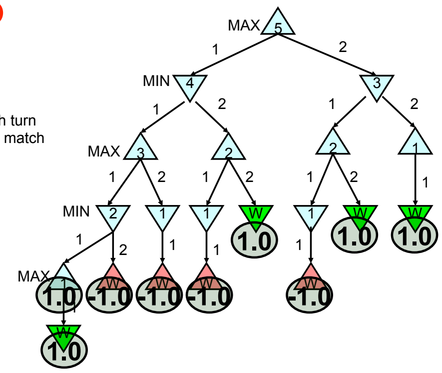

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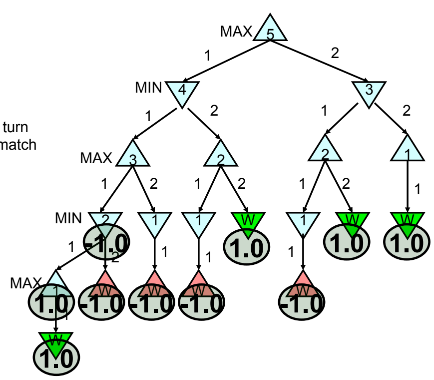

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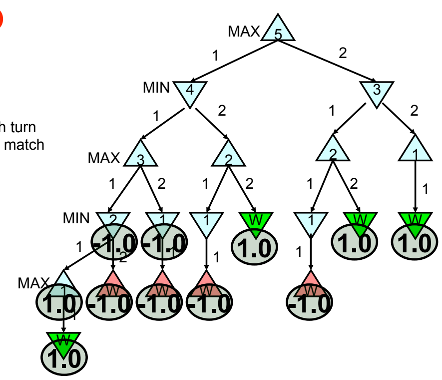

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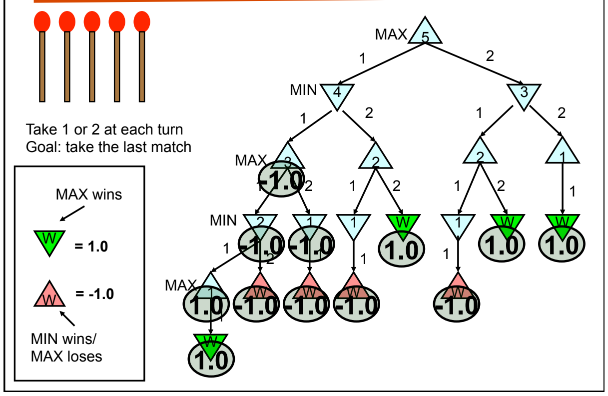

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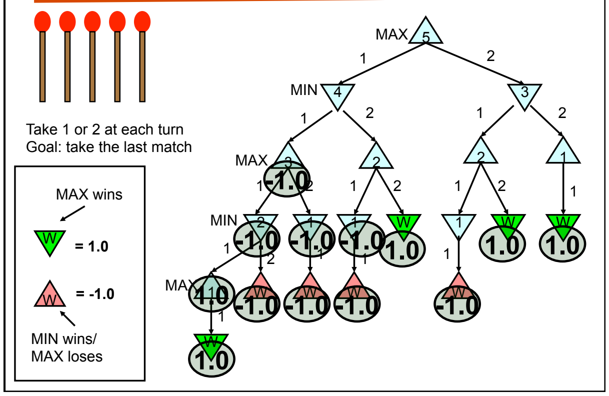

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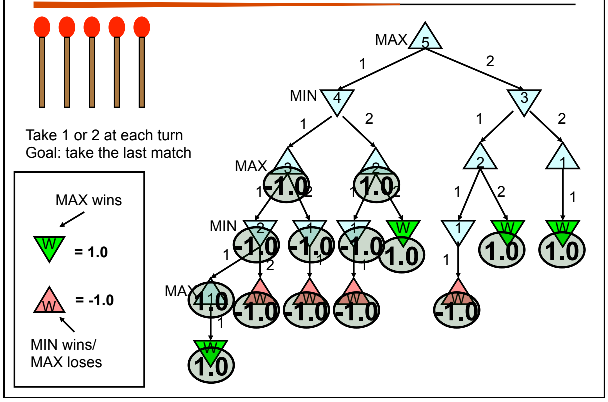
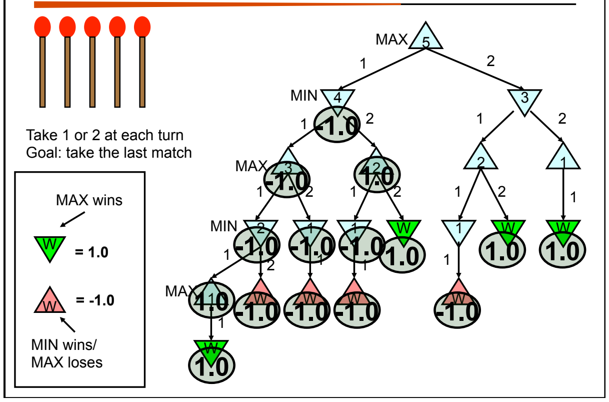
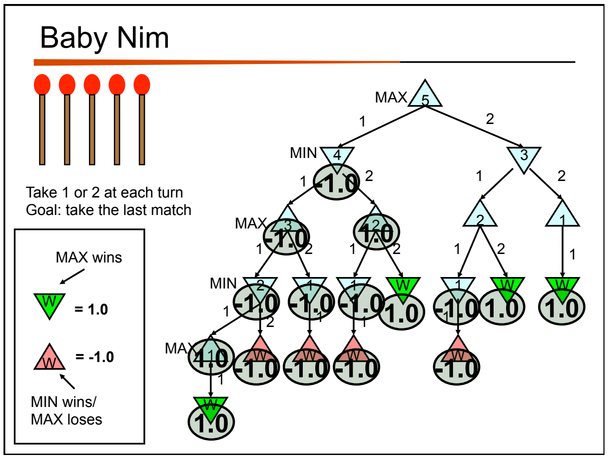
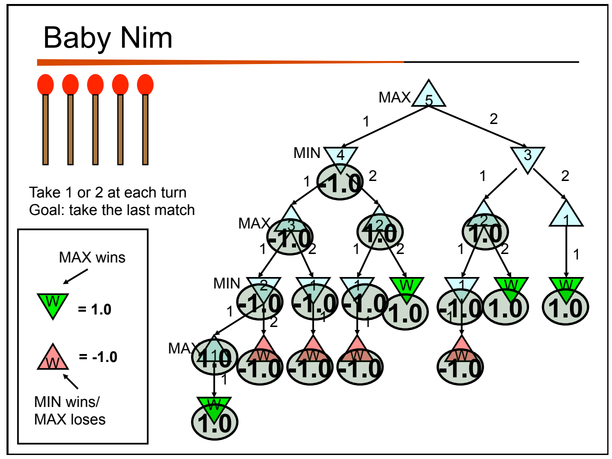


Baby Nim




Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses
MAX
MIN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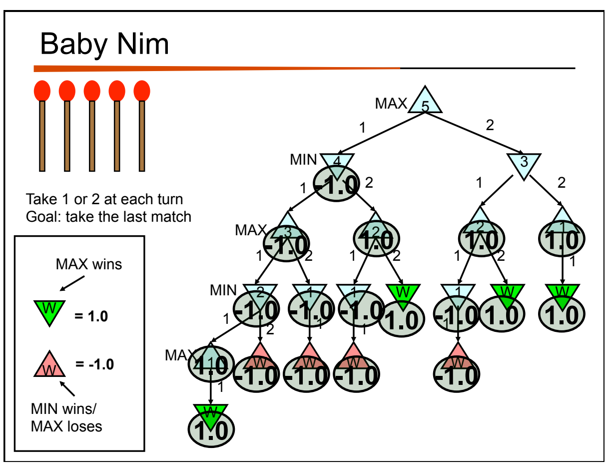
Baby Nim
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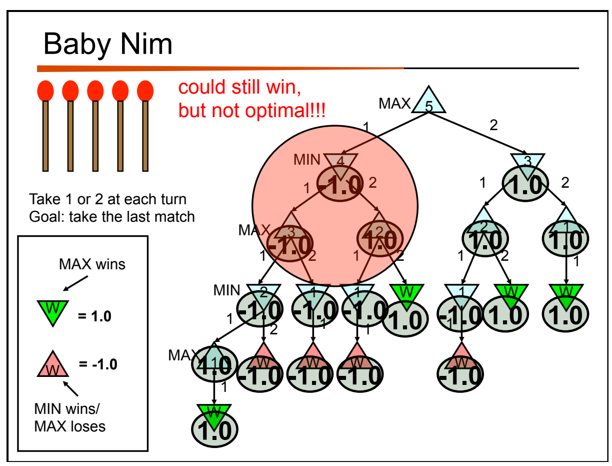
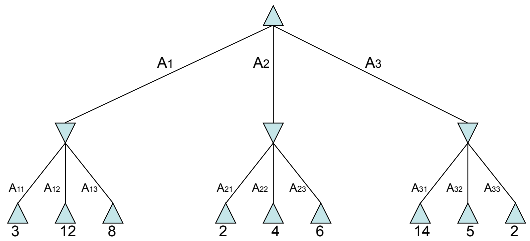
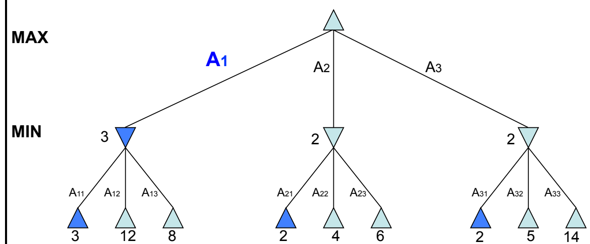

Baby Nim
could still win,
but not optimal!!!
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses
MAX
MIN

Minimax example 2



Which move should be made: A_1, A_2 or A_3?

Minimax example 2



Properties of minimax

Minimax is optimal!

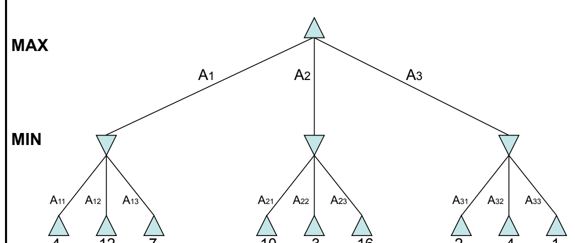
Are we done?

- For chess, b ≈ 35, d ≈100 for reasonable games → exact solution completely infeasible
- Is minimax feasible for Mancala or Tic Tac Toe?
  - Mancala: 6 possible moves. average depth of 40, so 6^{40} which is on the edge
  - Tic Tac Toe: branching factor of 4 (on average) and depth of 9… yes!

Ideas?

- pruning!
- improved state utility/evaluation functions

Pruning: do we have to traverse the whole tree?



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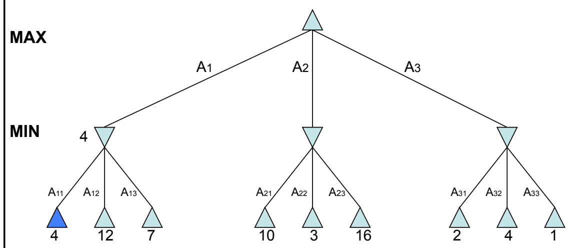
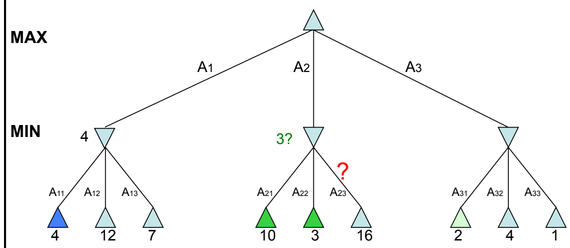
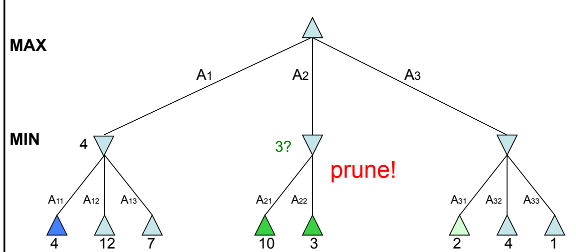


Minimax example 2

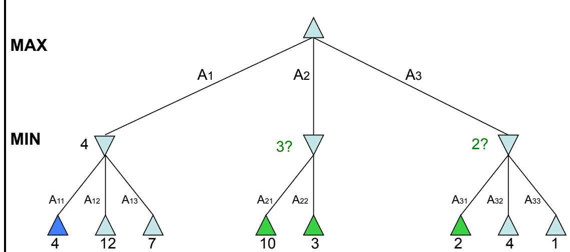


Minimax example 2



Any others if we continue?

Minimax example 2



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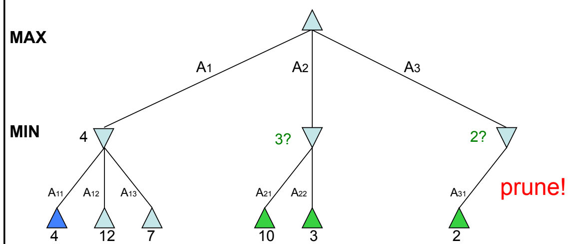


Alpha-Beta pruning

An optimal pruning strategy

- only prunes paths that are suboptimal (i.e. wouldn't be chosen by an optimal playing player)
- returns the *same* result as minimax, but faster

As we go, keep track of the best and worst along a path

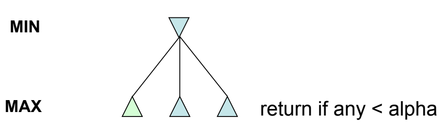
- alpha = best choice we've found so far for MAX
- beta = best choice we've found so far for MIN

Alpha-Beta pruning

alpha = best choice we've found so far for MAX

Using alpha and beta to prune:

- We're examining MIN's options for a ply
- To do this, we're examining all possible moves for MAX. If we find a value for one of MAX's moves that is **less than alpha**, return. **(MIN could do better down this path)**

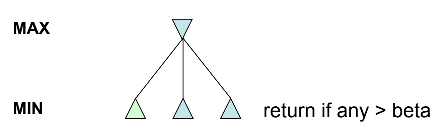


Alpha-Beta pruning

beta = best choice we've found so far for MIN

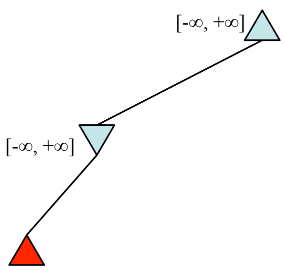
Using alpha and beta to prune:

- We're examining MAX's options for a ply
- To do this, we're examining all possible moves for MIN. If we find a value for one of MIN's possible moves that is **greater than beta**, return. **(MIN won't end up down here)**



Alpha-Beta pruning

Do DFS until we reach a leaf:



Alpha-Beta pruning

alpha = best choice we've found so far for MAX
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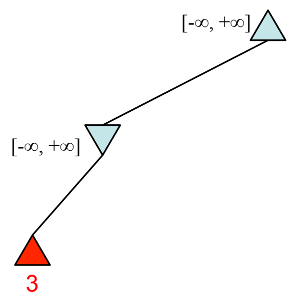
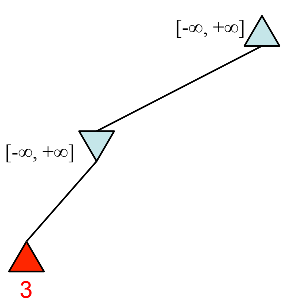


What do we know?

Alpha-Beta pruning

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



What do we know?

Alpha-Beta pruning

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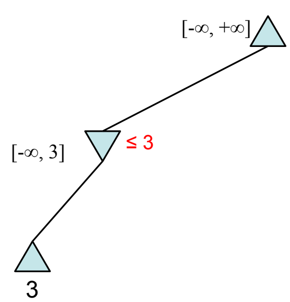
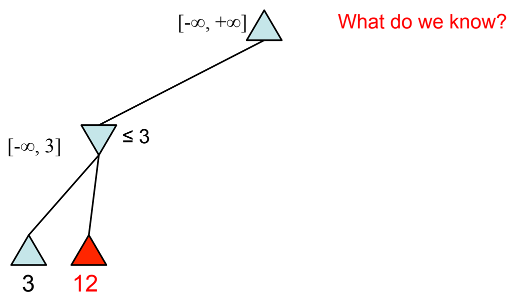


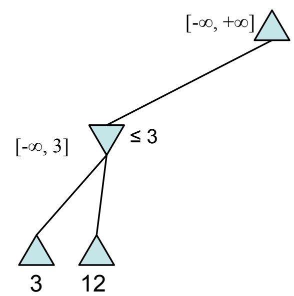
Alpha-Beta pruning

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



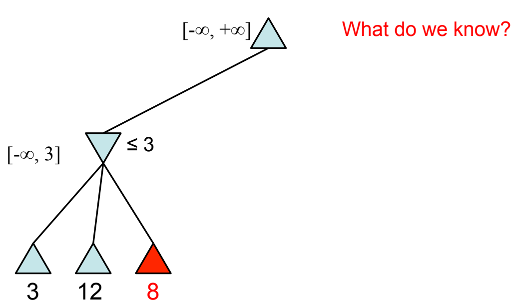
Alpha-Beta pruning

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



Alpha-Beta pruning

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



Alpha-Beta pruning

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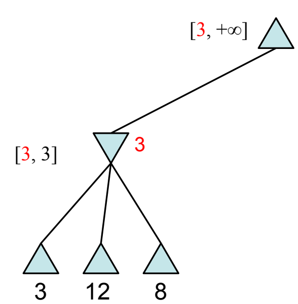
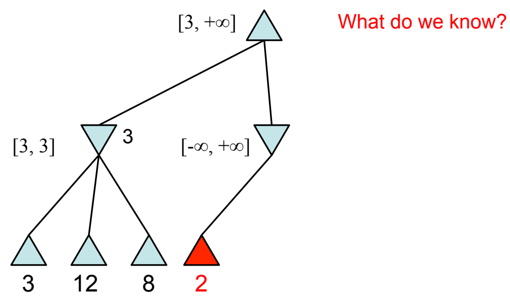
beta = best choice we've found so far for MIN



Alpha-Beta pruning

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



Alpha-Beta pruning

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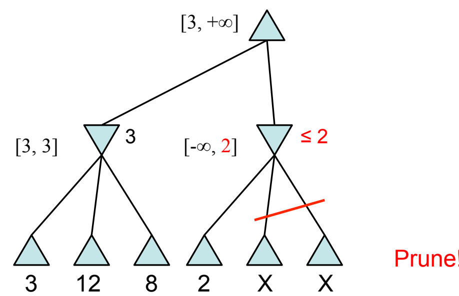

Alpha-Beta pruning

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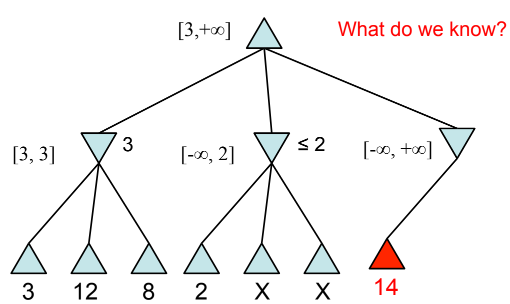

Alpha-Beta pruning

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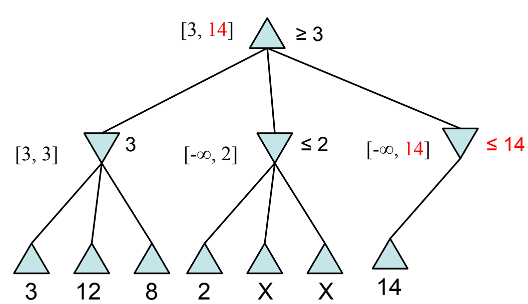
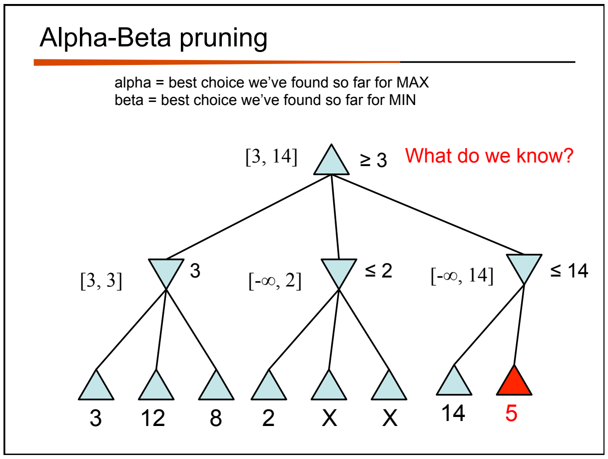
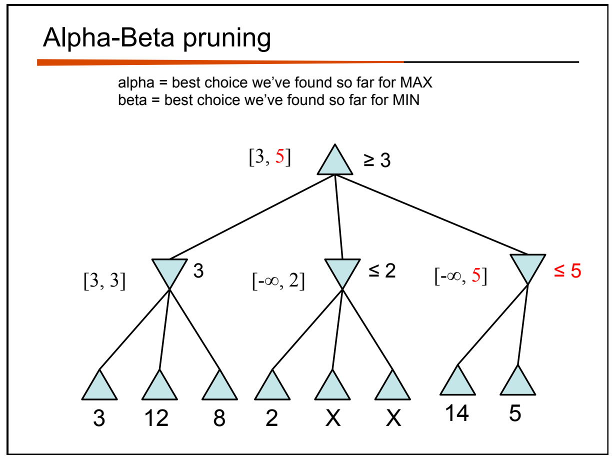
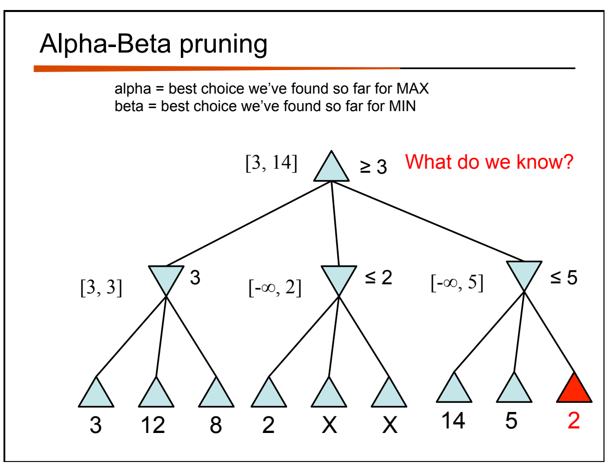
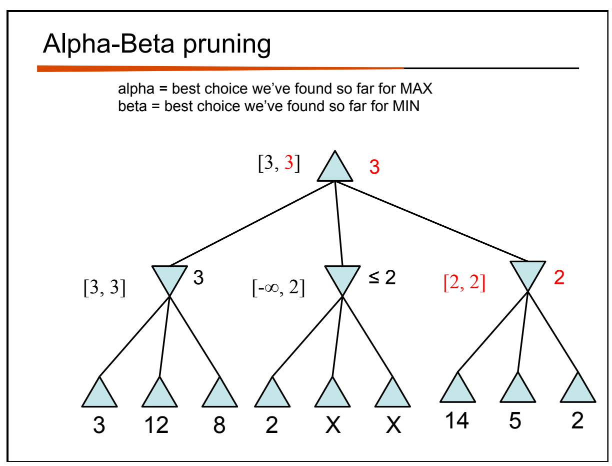



Alpha-Beta pruning
alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN
[3, 14]
≥ 3
What do we know?
[3, 3]
3
[-∞, 2]
≤ 2
[-∞, 14]
≤ 14
3
12
8
2
X
X
14
5


Alpha-Beta pruning
alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN
[3, 5]
≥ 3
[3, 3]
3
[-∞, 2]
≤ 2
[-∞, 5]
≤ 5
3
12
8
2
X
X
14
5


Alpha-Beta pruning
alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN
[3, 14]
≥ 3
What do we know?
[3, 3]
3
[-∞, 2]
≤ 2
[-∞, 5]
≤ 5
3
12
8
2
X
X
14
5
2


Alpha-Beta pruning
alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN
[3, 3]
3
[3, 3]
3
[-∞, 2]
≤ 2
[2, 2]
2
3
12
8
2
X
X
14
5
2

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN

```
def maxValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = -∞
    for all actions a in actions(state):
      value = max(value, minValue(result(state,a), alpha, beta)
      if value >= beta:
        return value # prune!
      alpha = max(alpha, value) # update alpha
    return value
```

We're making a decision for MAX.
- When considering MIN's choices, if we find a value that is greater than beta, stop, because MIN won't make this choice
- if we find a better path than alpha, update alpha

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN

```
def minValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = +∞
    for all actions a in actions(state):
      value = min(value, maxValue(result(state,a), alpha, beta)
      if value <= alpha:
        return value # prune!
      beta = min(beta, value) # update alpha
    return value
```

We're making a decision for MIN.
- When considering MAX's choices, if we find a value that is less than alpha, stop, because MAX won't make this choice
- if we find a better path than beta for MIN, update beta

Baby NIM2: take 1, 2 or 3 sticks

Effectiveness of pruning

Notice that as we gain more information about the state of things, we're more likely to prune

What affects the performance of pruning?
- key: which order we visit the states
- can try and order them so as to improve pruning

Effectiveness of pruning

If perfect state ordering:

- $O(b^m)$ becomes $O(b^{m/2})$
- We can solve a tree twice as deep!

Random order:

- $O(b^m)$ becomes $O(b^{3m/4})$
- still pretty good

For chess using a basic ordering

- Within a factor of 2 of $O(b^{m/2})$