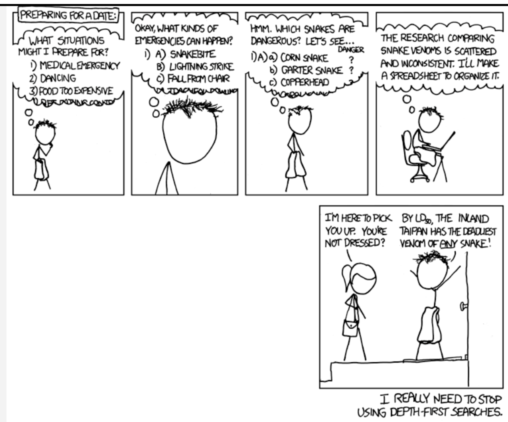


http://xkcd.com/761/

More Adversarial Search

CS311
David Kauchak
Spring 2013

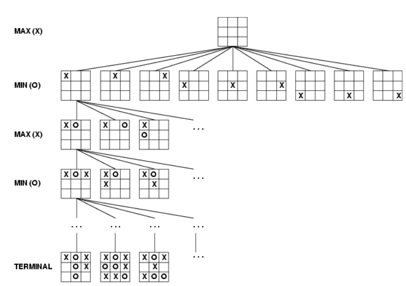
Some material borrowed from :
Sara Owsley Sood and others

Admin

- Written 2 posted today
- Assignment 2
  - Last chance for a partner
  - How's it going?
  - If working with a partner, should *both* be there when working on it!

Last time

Game playing as search



Last time

Game playing as search

Assume the opponent will play optimally
- MAX is trying to maximize utility
- MIN is trying to minimize utility

MINIMAX algorithm backs-up the value from the leaves by alternatively minimizing and maximizing action options
- plays "optimally", that is can play no better than

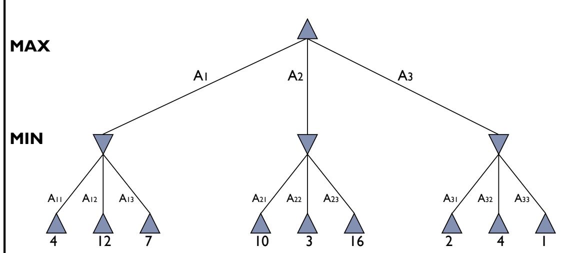
Won't work for deep trees or trees with large branching factors

Last time

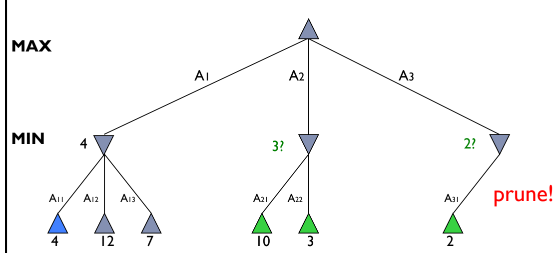
Pruning alleviates this be excluding paths

Alpha-Beta pruning retains the optimality, by pruning paths that will never be chosen
- alpha is the best choice down this path for MAX
- beta is the best choice down this path for MIN

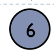
Pruning: do we have to traverse the whole tree?



Minimax example 2



Baby NIM2: take 1, 2 or 3 sticks



alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN

```
def maxValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = -∞
    for all actions a in actions(state):
      value = max(value, minValue(result(state,a), alpha, beta)
      if value >= beta:
        return value # prune!
      alpha = max(alpha, value) # update alpha
    return value
```

We're making a decision for MAX.
- When considering MIN's choices, if we find a value that is greater than beta, stop, because MIN won't make this choice
- if we find a better path than alpha, update alpha

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN

```
def minValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = +∞
    for all actions a in actions(state):
      value = min(value, maxValue(result(state,a), alpha, beta)
      if value <= alpha:
        return value # prune!
      beta = min(beta, value) # update alpha
    return value
```

We're making a decision for MIN.
- When considering MAX's choices, if we find a value that is less than alpha, stop, because MAX won't make this choice
- if we find a better path than beta for MIN, update beta

Effectiveness of alpha-beta pruning

As we gain more information about the state of things, we're more likely to prune

What affects the performance of pruning?
- key: which order we visit the states
- can try and order them so as to improve pruning

Effectiveness of pruning

If perfect state ordering:

- $O(b^m)$ becomes $O(b^{m/2})$
- We can solve a tree twice as deep!

Random order:

- $O(b^m)$ becomes $O(b^{3m/4})$
- still pretty good

For chess using a basic ordering

- Within a factor of 2 of $O(b^{m/2})$

Evaluation functions

$O(b^{m/2})$ is still exponential (and that's assuming optimal pruning)

- for chess, this gets us ~10-14 ply deep (a bit more with some more heuristics)
  - 200 million moves per second (on a reasonable machine)
  - 35^5 = 50 million, or < 1 second
- not enough to solve most games!

Ideas?

- heuristic function – evaluate the desirability of the position
- This is not a new idea:
  - Claude Shannon (think-- information theory, entropy), "Programming a Computer for Playing Chess" (1950)
  - http://vision.unipv.it/IA1/ProgrammingaComputerforPlayingChess.pdf
    - page 3
    - page 5

Cutoff search

How does an evaluation function help us?

- search until some stopping criterion is met
- *return our heuristic evaluation of the state at that point*
  - This serves as a **proxy** for the actual scoring function

When should we stop?

- as deep as possible, for the time constraints
- generally speaking, the further we are down the tree, the more accurate our evaluation function will be
- based on a fixed depth
  - keep track of our depth during recursion
  - if we reach our depth limit, return EVAL(state)

Cutoff search

When should we stop?

- fixed depth
- based on time
  - start a timer and run IDS
  - when we run out of time, return the result from the last completed depth
- quiescence search
  - search using one of the cutoffs above
  - but if we find ourselves in a volatile state (for example a state where a piece is about to be captured) keep searching!
  - attempts to avoid large swings in EVAL scores

Heuristic EVAL

What is the goal of EVAL, our state evaluation function?

- estimate the *expected* utility of the game at a given state

What are some requirements?

- must be efficient (we're going to be asking this about a lot of states)
- EVAL should play nice with terminal nodes
  - it should order terminal nodes in the same order at UTILITY
  - a win should be the most desirable thing
  - a loss should be the least desirable thing

Heuristic EVAL

What are some desirable properties?

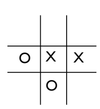
- value should be higher the closer we are to a win
- and lower the closer we are to a lose

The quality of the evaluation function impacts the quality of the player

- Remember last time (De Groot), we expert players were good at evaluating board states!

Simple Mancala Heuristic: Goodness of board = # stones in my Mancala minus the number of stones in my opponents.

Tic Tac Toe evaluation functions



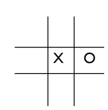
Ideas?

Example Tic Tac Toe EVAL

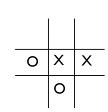
Tic Tac Toe
Assume MAX is using "X"

EVAL(*state*) =

if state is win for MAX:
    $+\infty$
if state is win for MIN:
    $-\infty$
else:
    (number of rows, columns and diagonals available to MAX) - (number of rows, columns and diagonals available to MIN)

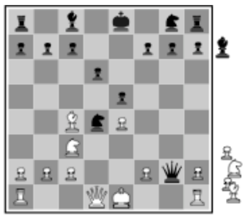


= 6 - 4 = 2



= 4 - 3 = 1

Chess evaluation functions



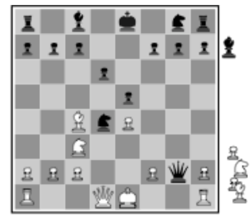
Ideas?

Chess EVAL

Assume each piece has the following values

pawn = 1;
knight = 3;
bishop = 3;
rook = 5;
queen = 9;

EVAL(state) =
sum of the value of white pieces –
sum of the value of black pieces



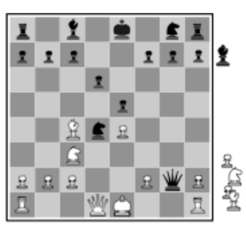
= 31 - 36 = -5

Chess EVAL

Assume each piece has the following values

pawn = 1;
knight = 3;
bishop = 3;
rook = 5;
queen = 9;

EVAL(state) =
sum of the value of white pieces –
sum of the value of black pieces



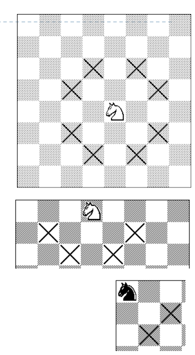
Any problems with this?

Chess EVAL

Ignores actual positions!

Actual heuristic functions are often a weighted combination of features

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$



Chess EVAL

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$

number of pawns

number of attacked knights

1 if king has knighted, 0 otherwise

A feature can be any numerical information about the board

- as general as the number of pawns
- to specific board configurations

Deep Blue: 8000 features!

Chess EVAL

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$

number of pawns

number of attacked knights

1 if king has knighted, 0 otherwise

How can we determine the weights (especially if we have 8000 of them!)?

Chess EVAL

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$

number of pawns

number of attacked knights

1 if king has knighted, 0 otherwise

Machine learning!
- play/examine lots of games
- adjust the weights so that the EVAL(s) correlates with the actual utility of the states

Horizon effect

Black to move

Who's ahead? What move should Black make?

Horizon effect

The White pawn is about to become a queen

A naïve EVAL function may not account for this behavior

We can delay this from happening for a long time by putting the king in check

If we do this long enough, it will go beyond our search cutoff and it will appear to be a **better** move than any move allowing the pawn to become a queen

But it's only *delaying the inevitable* (the book also has another good example)

Black to move

Other improvements

Computers have lots of memory these days

DFS (or IDS) is only using a linear amount of memory

How can we utilize this extra memory?

- transposition table
- history/end-game tables
- "opening" moves
- …

Transposition table

Similar to keeping track of the list of explored states

- "transpositions" are differing move sequences that start and end in the same place

Keeps us from duplicating work

Can double the search depth in chess!

history/end-game tables

History

- keep track of the quality of moves from previous games
- use these instead of search

end-game tables

- do a reverse search of certain game configurations, for example all board configurations with king, rook and king
- tells you what to do in ***any*** configuration meeting this criterion
- if you ever see one of these during search, you lookup exactly what to do

end-game tables

Devastatingly good

Allows much deeper branching

- for example, if the end-game table encodes a 20-move finish and we can search up to 14
- can search up to depth 34

Stiller (1996) explored all end-games with 5 pieces

- one case check-mate required 262 moves!

Knoval (2006) explored all end-games with 6 pieces

- one case check-mate required 517 moves!

Traditional rules of chess require a capture or pawn move within 50 or it's a stalemate

Opening moves

At the very beginning, we're the farthest possible from any goal state

People are good with opening moves

Tons of books, etc. on opening moves

Most chess programs use a database of opening moves rather than search

Chance/non-determinism in games

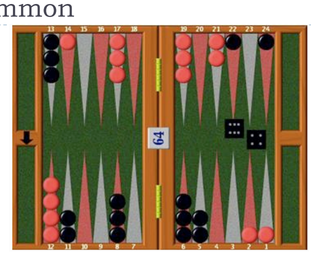
All the approaches we've looked at are only appropriate for deterministic games

Some games have a randomness component, often imparted either via dice or shuffling

Why consider games of chance?

- because they're there!
- more realistic… life is not deterministic
- more complicated, allowing us to further examine search techniques

Backgammon



Basic idea: move your pieces around the board and then off

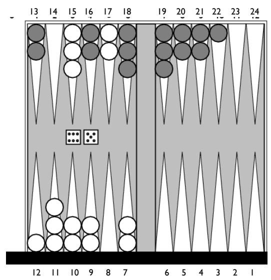
Amount you get to move is determined by a roll of two dice

Backgammon

If we know the dice rolls, then it's straightforward to get the next states

For example, white rolls a 5 and a 6

Possible moves?

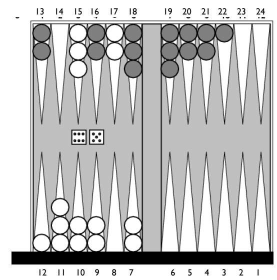


Backgammon

If we know the dice rolls, then it's straightforward to get the next states

For example, white rolls a 5 and a 6

Possible moves (7-2,7-1), (17-12,17-11), …

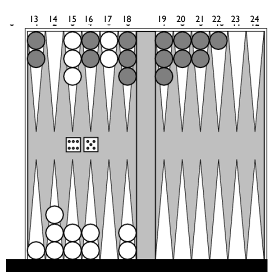


Backgammon

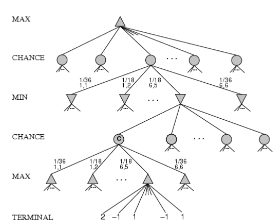
Which is better: (7-2,7-1) or (17-12,17-11)?

We'd like to search…

Ideas?

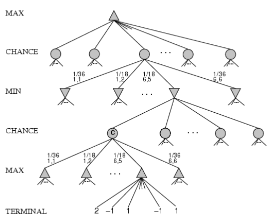


Searching with chance



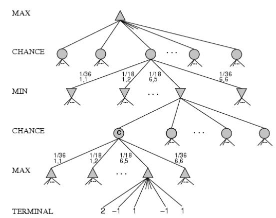
- We know there are 36 different dice rolls (21 unique)
- Insert a "chance" layer in each ply with branching factor 21
- What does this do to the branching factor?
  - drastically increases the branching factor (by a factor of 21!)

Searching with chance



Are all dice combinations equally likely?

Searching with chance



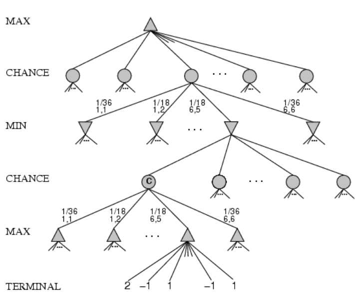
Associate a probability with each chance branch

each double ([1,1], [2,2], …) have probability 1/36

all others have probability 1/18

Generally the probabilities are easy to calculate

Searching with chance



Assume we can reach the bottom.

How can we calculate the value of a state?

Expected minimax value

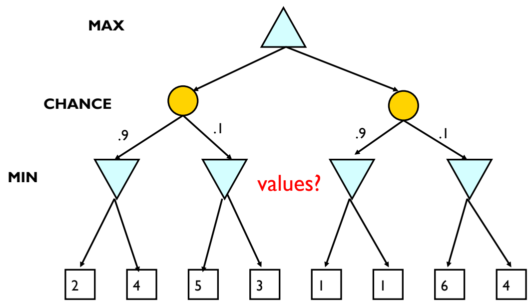
Rather than the actual value calculate the expected value based on the probabilities

EXPECTI-MINIMAX-VALUE(n)=

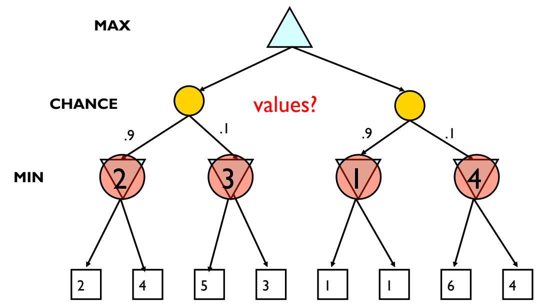
UTILITY(n)	If n is a terminal
$\max_{s \in successors(n)}$ MINIMAX-VALUE(s)	If n is a max node
$\min_{s \in successors(n)}$ MINIMAX-VALUE(s)	If n is a min node
$\sum_{s \in successors(n)} P(s)$. EXPECTIMINIMAX(s)	If n is a chance node

multiply the minimax value by the probability of going to that state

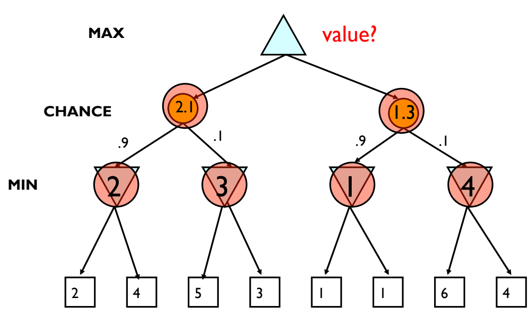
EXPECTEDMINIMAX example



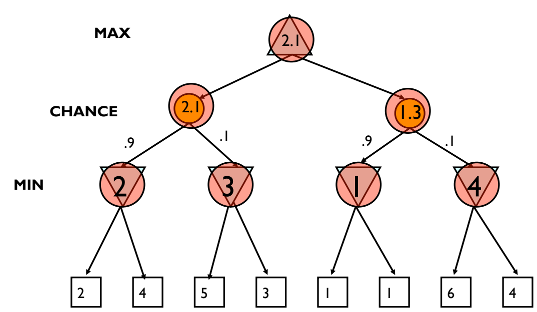
EXPECTEDMINIMAX example



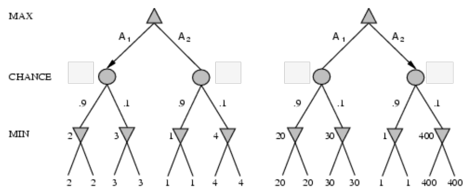
EXPECTEDMINIMAX example



EXPECTEDMINIMAX example



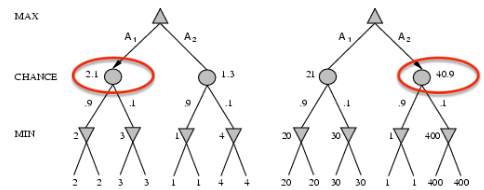
Chance and evaluation functions



Which move will be picked for these different MINIMAX trees?

Does this change any requirements on the evaluation function?

Chance and evaluation functions



Even though we haven't changed the ranking of the final states, we changed the outcome

Magnitude matters, not just ordering

Games with chance

Original branching factor *b*, chance factor *n*

What happens to our search run-time?

- $O((nb)^m)$
- in essence, multiplies our branching factor by n

For this reason, many games with chance don't use much search

- backgammon frequently only looks ahead 3 ply

Instead, evaluation functions play a more important roll

- TD-Gammon learned an evaluation function by playing itself over a million times!

Partially observable games

In many games we don't have all the information about the world, for example?

Partially observable games

In many games we don't have all the information about the world

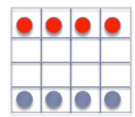
- battleship
- bridge
- poker
- scrabble
- Kriegspiel
  - pretty cool game
  - "hidden" chess
- …

How can we deal with this?

Simple Kriegspeil

To start with:

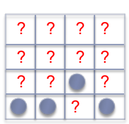
- I know where my pieces are
- and I know exactly where the opponents pieces are



Simple Kriegspeil

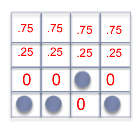
As the game progresses, though

- I know where my pieces are
- but I no longer know where the opponents pieces are



Simple Kriegspeil

However, I can have some expectation/estimation of where the are



starts to look like a game of chance

Challenges with partially observable games?

state space can be huge!

our MINIMAX assumption is probably not true

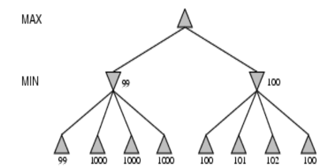
- reasons for the opponent to purposefully play suboptimally

may make moves just to explore

These are hard!

- when humans play Kriegspeil, most of the checkmates are accidental ☺

Other things to watch out for…



What will minimax do here?

Is that OK?

What might you do instead?

State of the art

5.7 of the book gives a pretty good recap of popular games

Still lots of research going on!

AAAI has an annual poker competition

Lots of other tournaments going on for a variety of games

New games being invented/examined all the time

- google "quantum chess"

University of Alberta has a big games group

- http://webdocs.cs.ualberta.ca/~games/