CS52 - Midterm 3 Sample Problems

This document contains some sample exercises. A few things to note:

- This is not a practice midterm; it is a collection of questions for you to practice.
- **The questions cover some of the topics that are on the exam, but may not be comprehensive.**
- Many of the individual problems are longer than would be on an actual midterm and some are more complicated.

Put another way, I'd strongly advise you to study first and then try and do these practice problems, rather than using the practice problems to guide your studying.

You are free to talk about the exercises with anyone, but they are perhaps best used in a *solo* test-like situation. A solution set will not be distributed.

1. Consider six-bit words using twos complement signed representation.
   (a) Give the decimal value and the bit pattern of the largest integer that can be represented in six bits.
   (b) Negate the value in part 1a. Express the result as a decimal value and a bit pattern.
   (c) Give the decimal value and the bit pattern of the smallest (most negative) integer that can be represented in six bits.
   (d) Negate the value in part 1c. Express the result as a decimal value and a bit pattern.
2. More number fun
   (a) Give an example of two six-bit words whose sum is correct when interpreted in both the unsigned and signed representations.
   (b) Give an example of two six-bit words whose sum is incorrect when interpreted in both the unsigned and signed representations.
   (c) Give an example of two six-bit words whose sum is correct when interpreted as an unsigned value but incorrect as a signed value.
   (d) Give an example of two six-bit words whose sum is correct when interpreted as a signed value but incorrect as an unsigned value.

3. One of our examples in class was a 2^n-to-1 multiplexer. Consider the reverse idea, a 1-to-2^n multiplexer. It has one input line, n control lines, and 2^n output lines. The control lines select one output line to have the value of the input line; all the other output lines have the value 0.

   (a) Draw a circuit diagram for a 1-to-2 multiplexer.

   (b) Using the 1-to-2 multiplexer as a component, draw a diagram for a 1-to-4 multiplexer.

4. Draw a simple circuit using gates to implement a circuit that has two inputs and two outputs and has the truth table shown below. (On a real midterm, there would be a table of gates.)

input		output	
x	y	o1	o2
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

5. We saw the polymorphic type `binTree`, which represented trees having nodes with at most two children. A 2-3 tree is a tree in which each non-leaf node may have either two or three children and all subtrees of a node have the same height.

   If we ignore the condition on the heights of subtrees, we can make the following SML type definition.

```
datatype 'a twoThreeTree =
  | Empty
  | Binary of 'a * 'a twoThreeTree * 'a twoThreeTree
  | Ternary of 'a * 'a twoThreeTree * 'a twoThreeTree * 'a twoThreeTree;
```

   (a) Write a recursive function N that computes the number of nodes in a 2-3 tree.

   (b) Write a recursive function ht that computes the height of a 2-3 tree. (In analogy with binary trees, make the height of the empty tree 1.)

   (c) What is the shortest 2-3 tree with 47 nodes? the tallest?

6. What is the mod-15 multiplicative inverse of 7, i.e. what is inverseMod 7 15?