

## CS52 RECURSION

David Kauchak  
CS 52 – Spring 2017

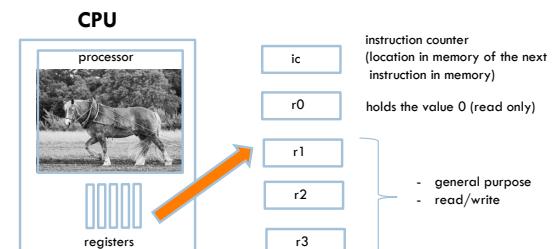
### Admin

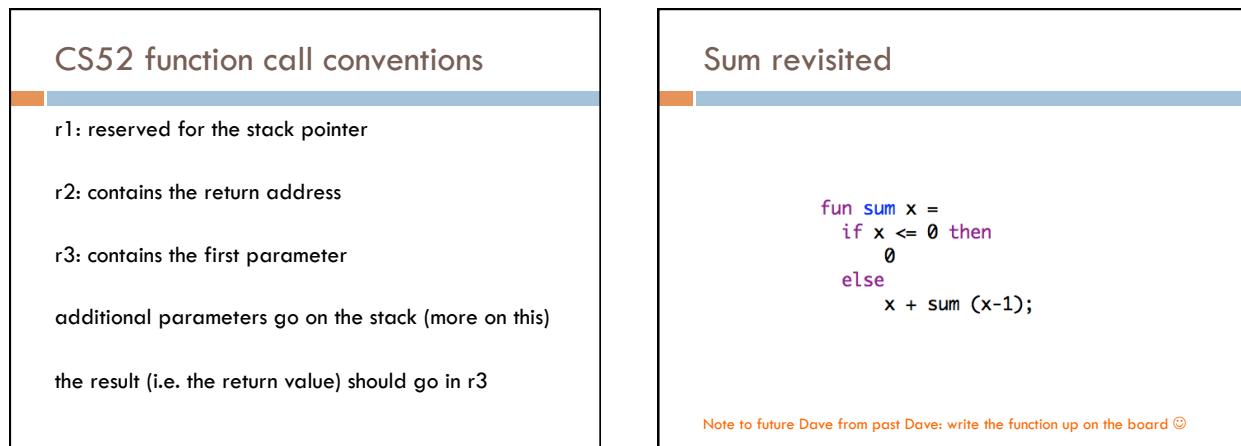
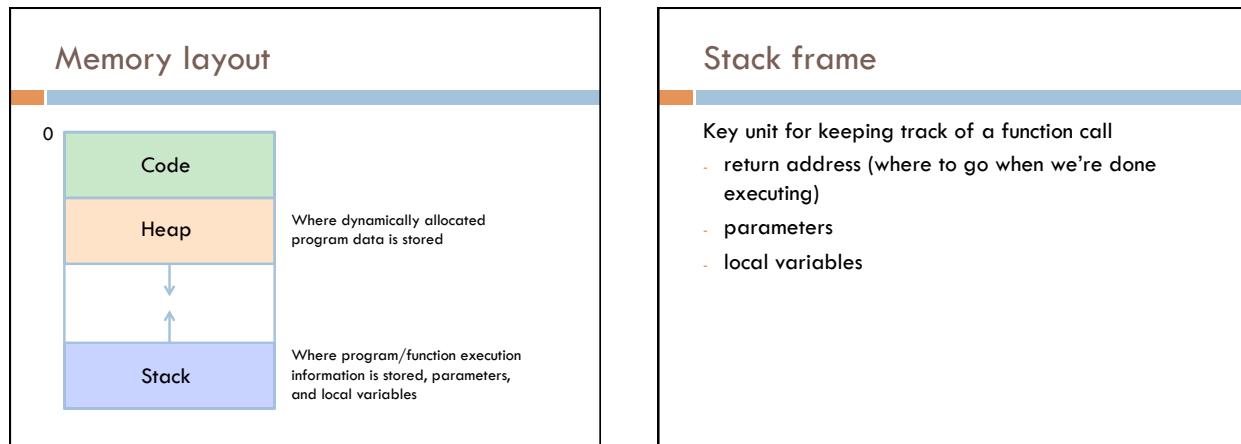
- Final grade
- Assignment grading
- Assignment 4
- Reading

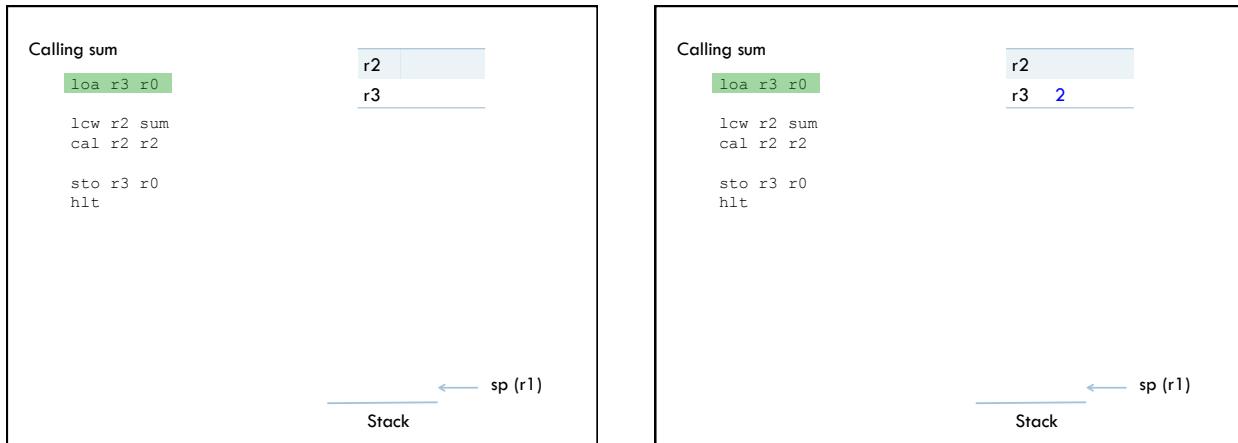
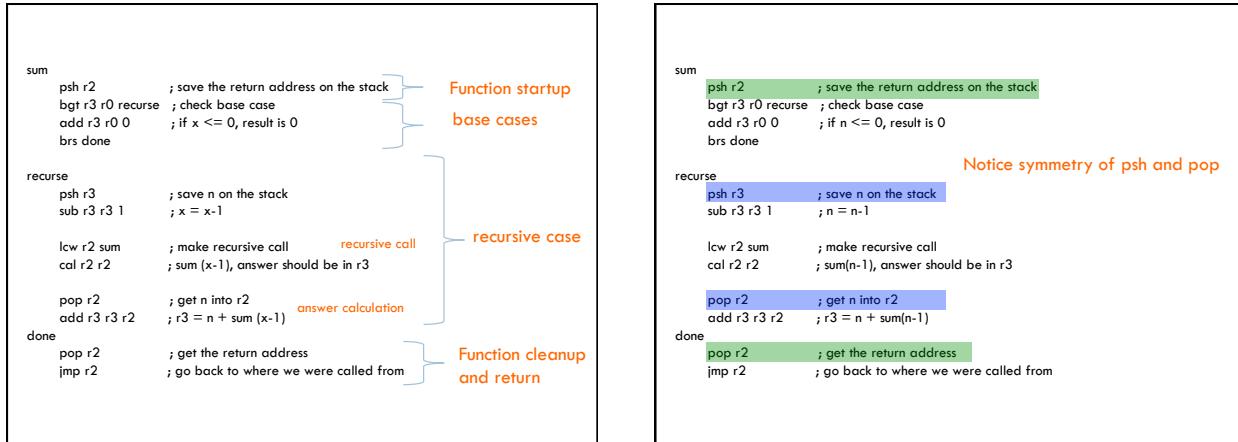
### Examples from this lecture

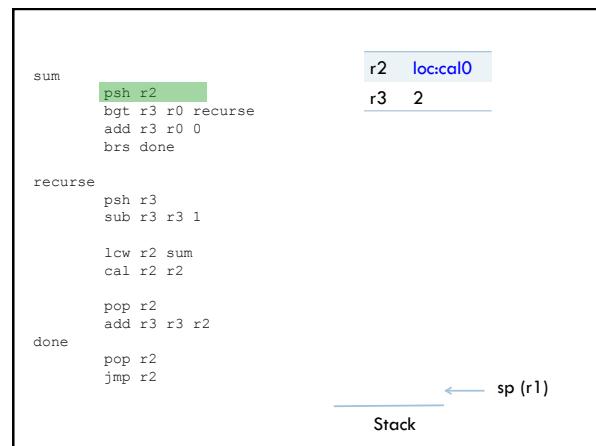
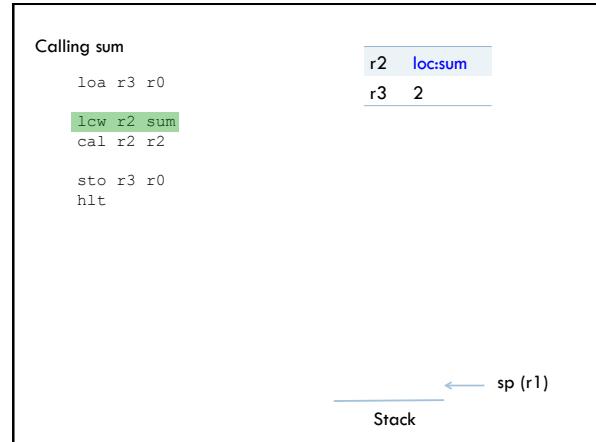
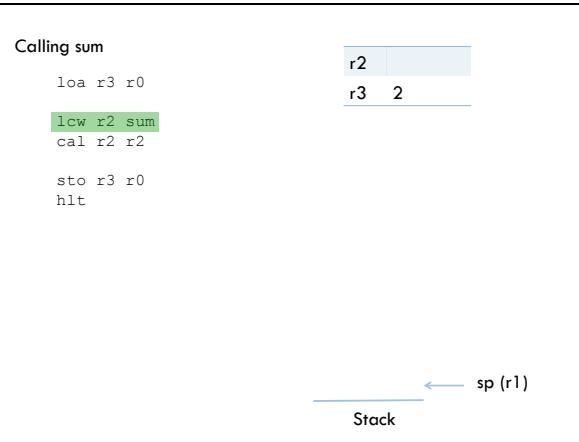
<http://www.cs.pomona.edu/~dkauchak/classes/cs52/examples/cs52machine/>

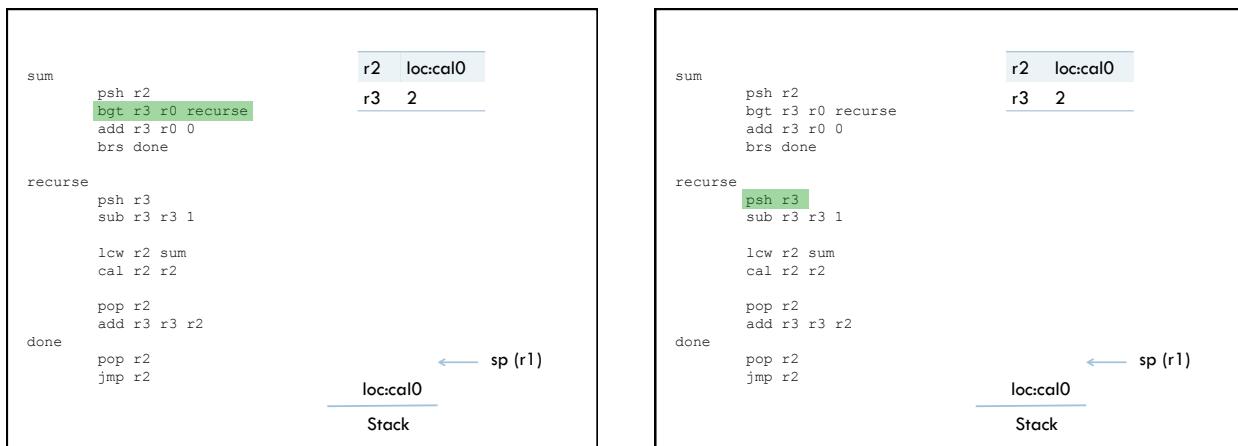
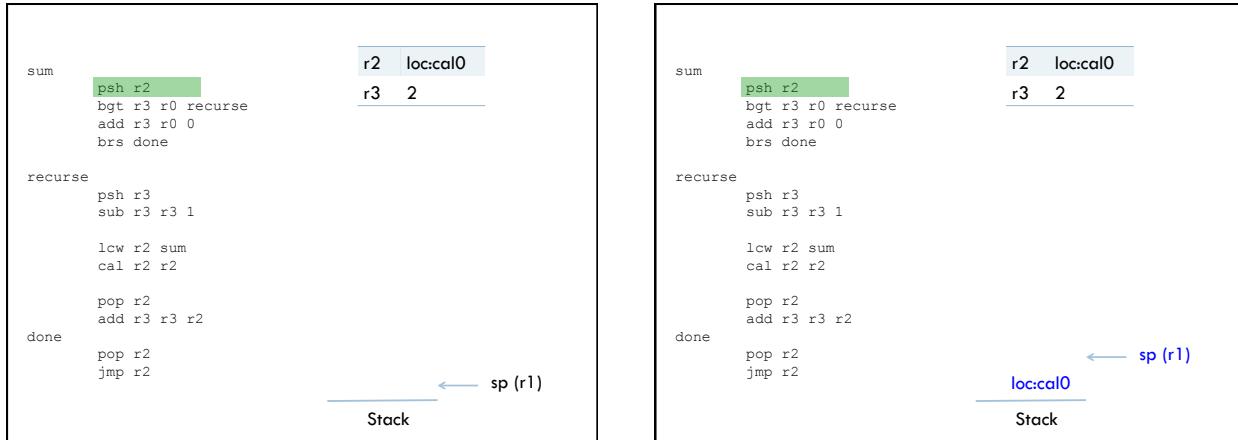
### CS52 machine

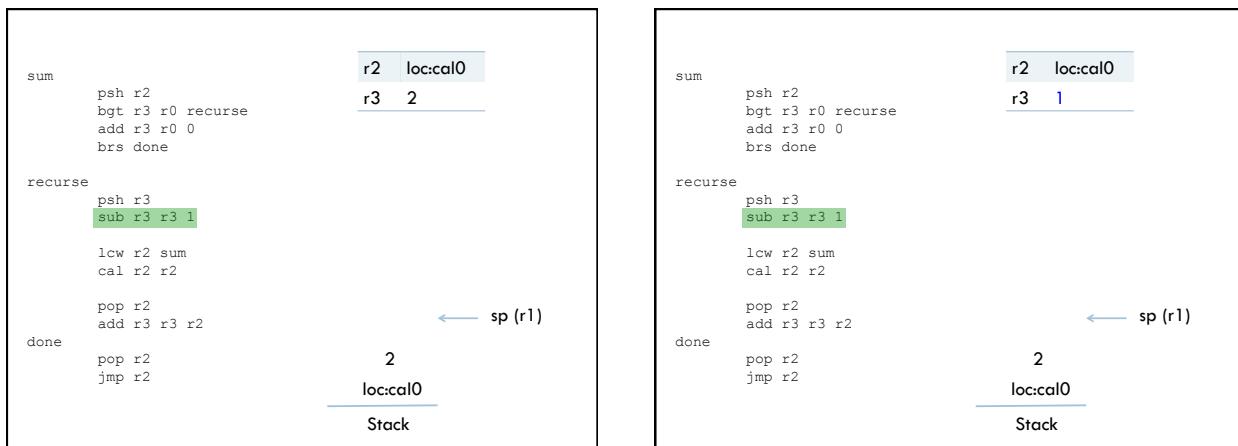
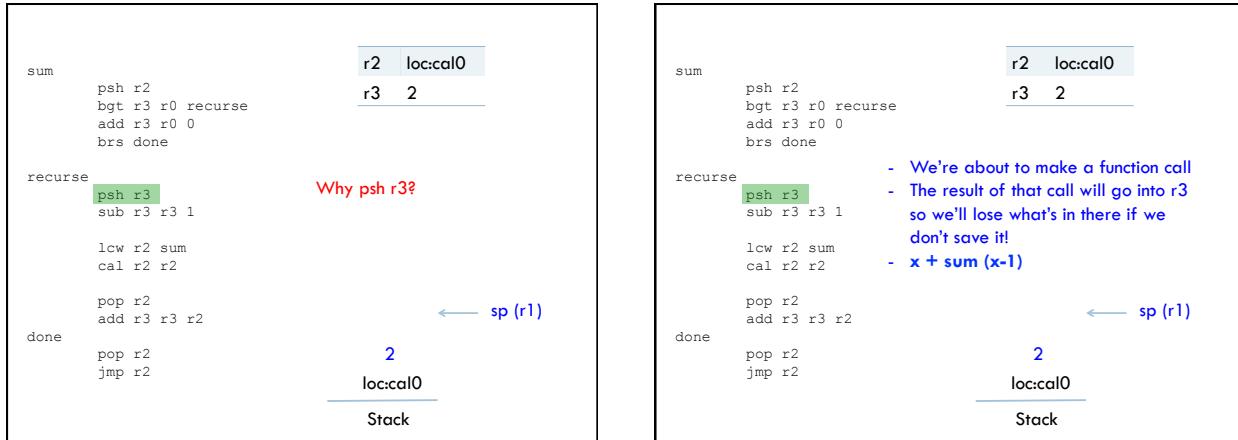


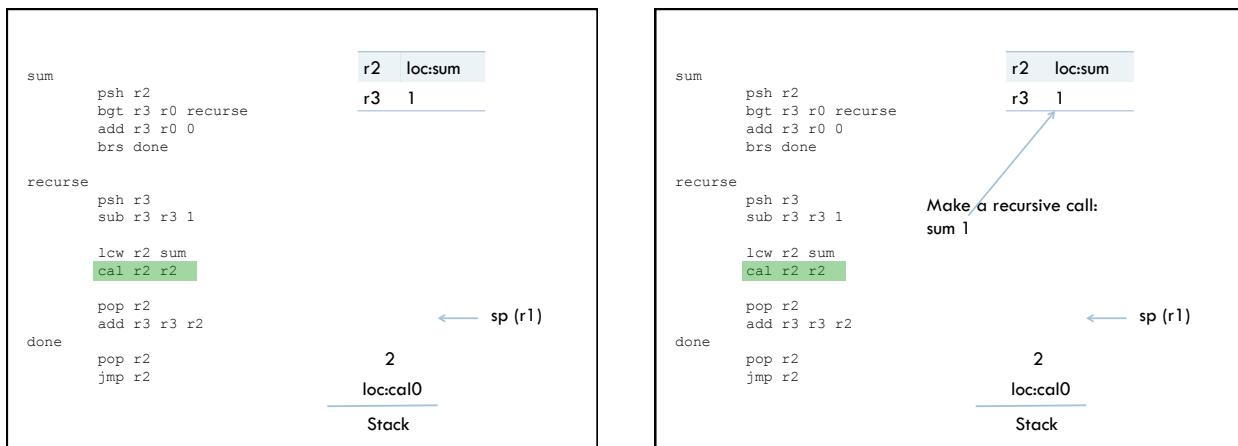
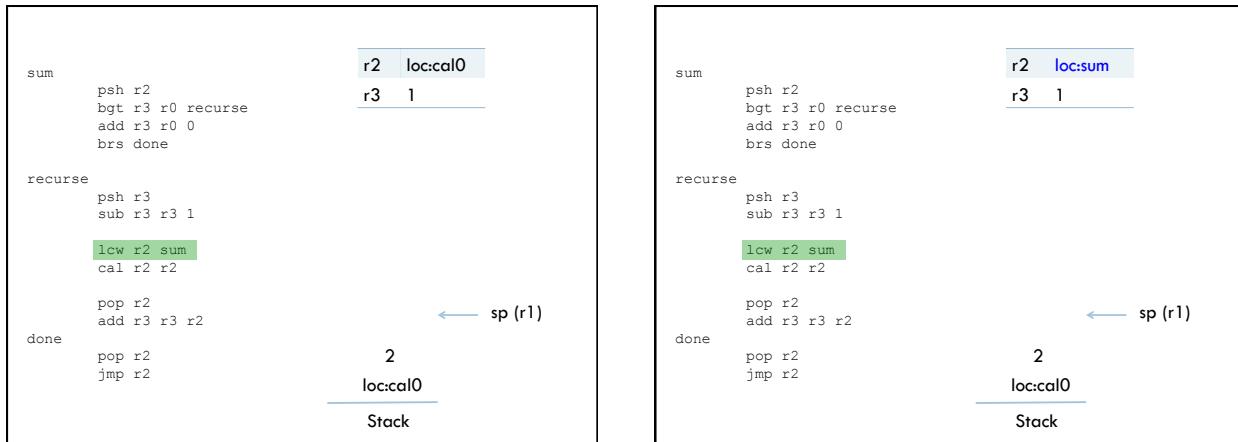


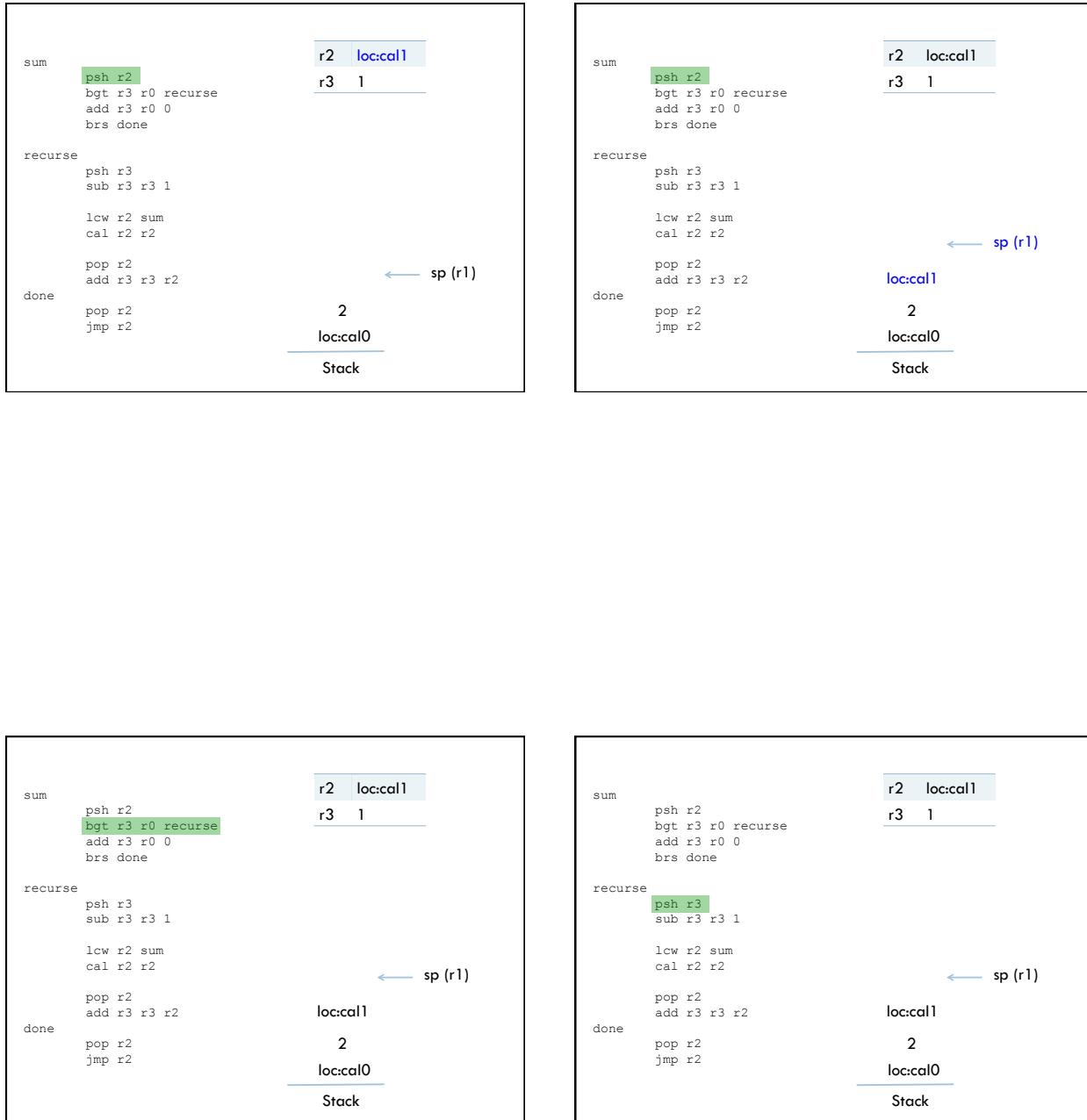


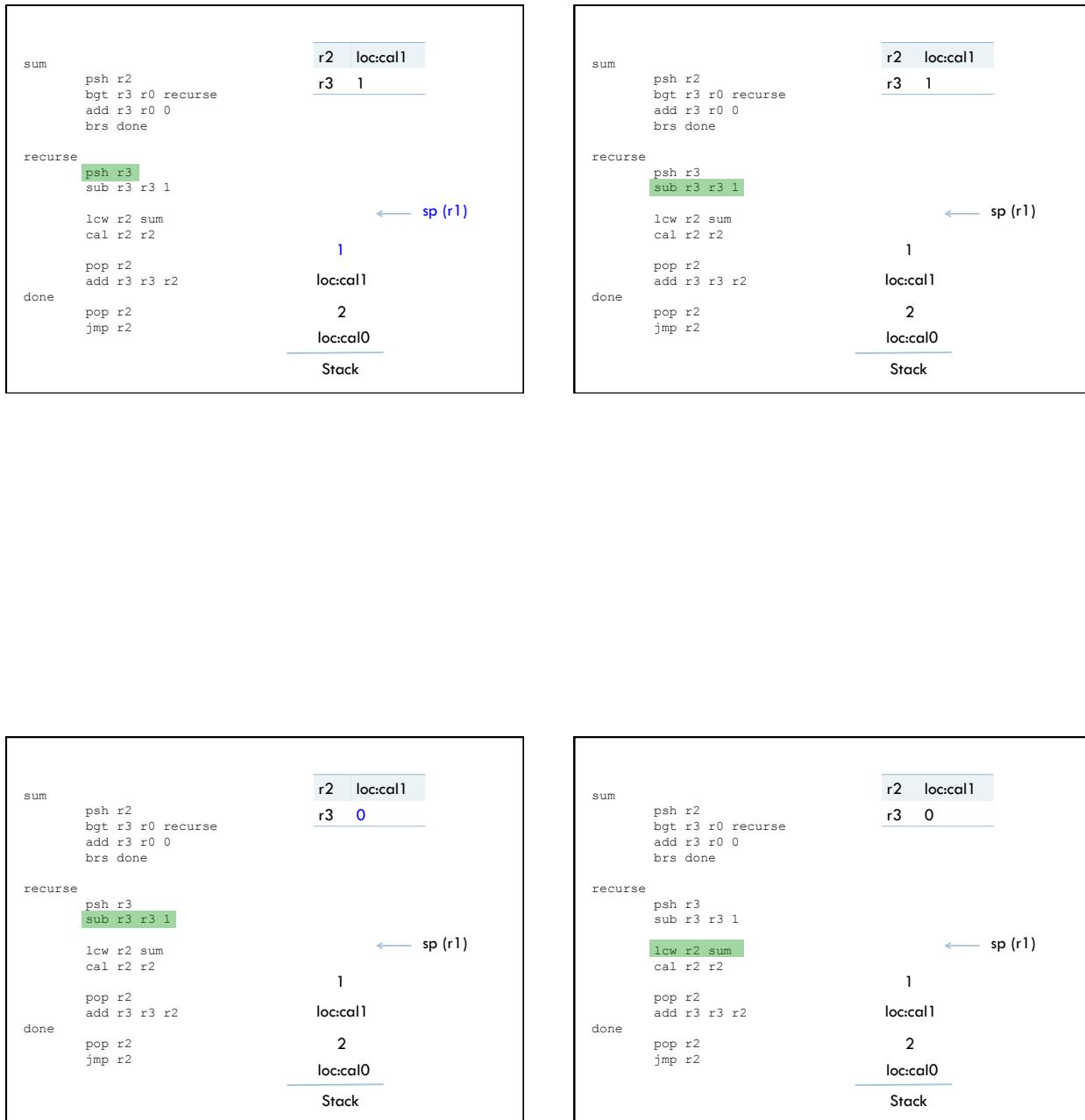


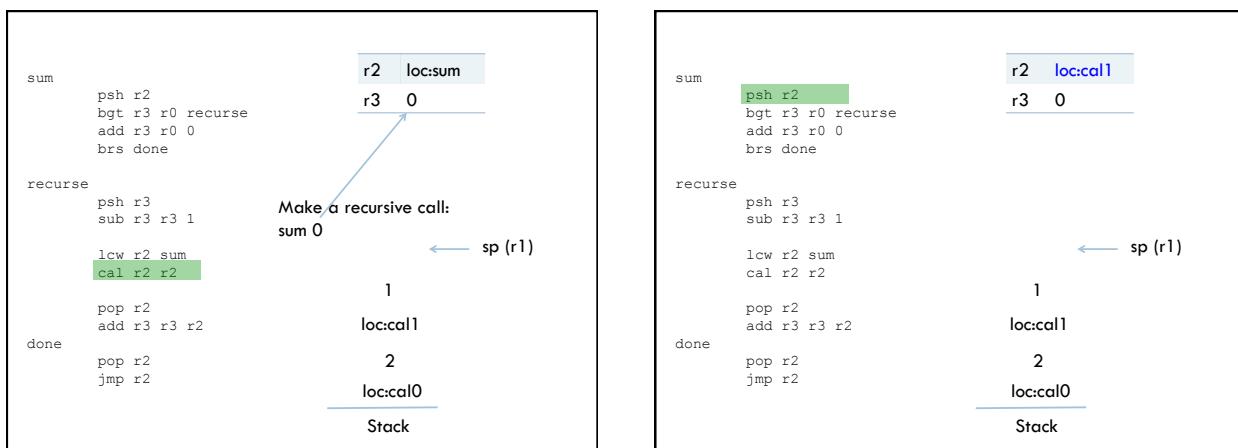
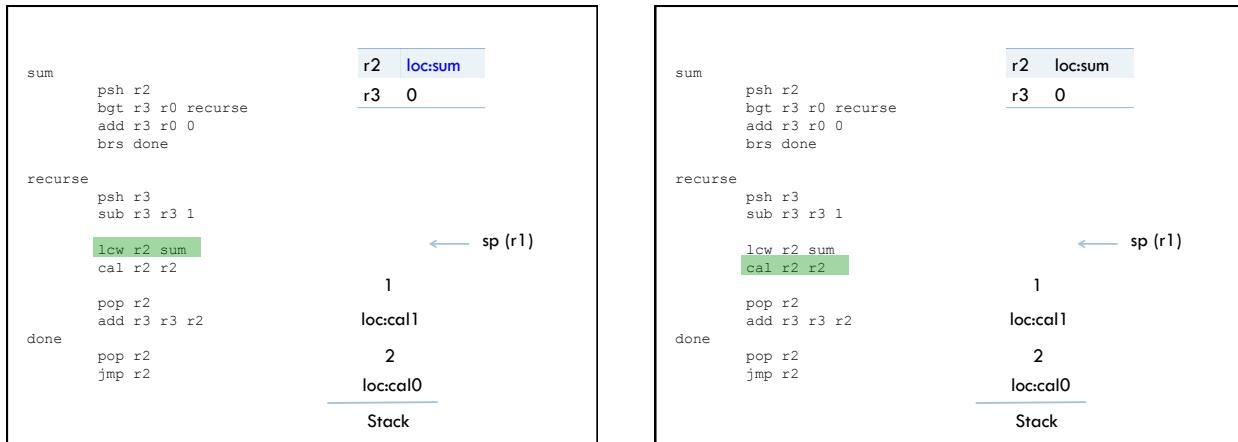


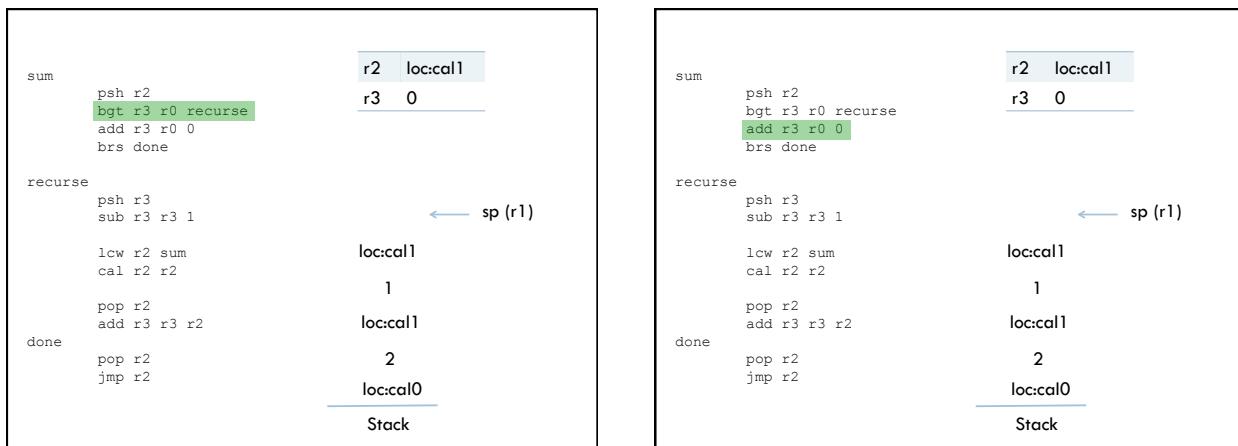
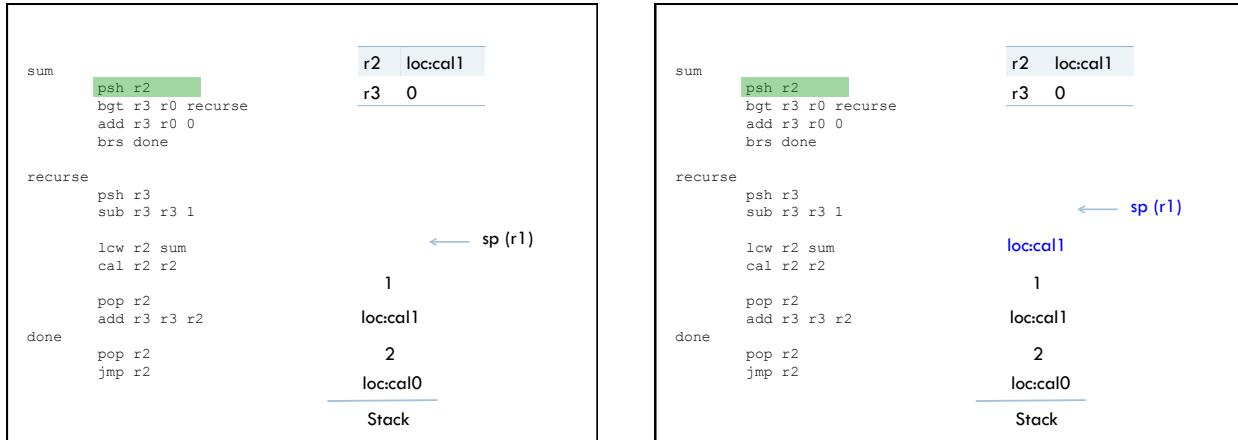


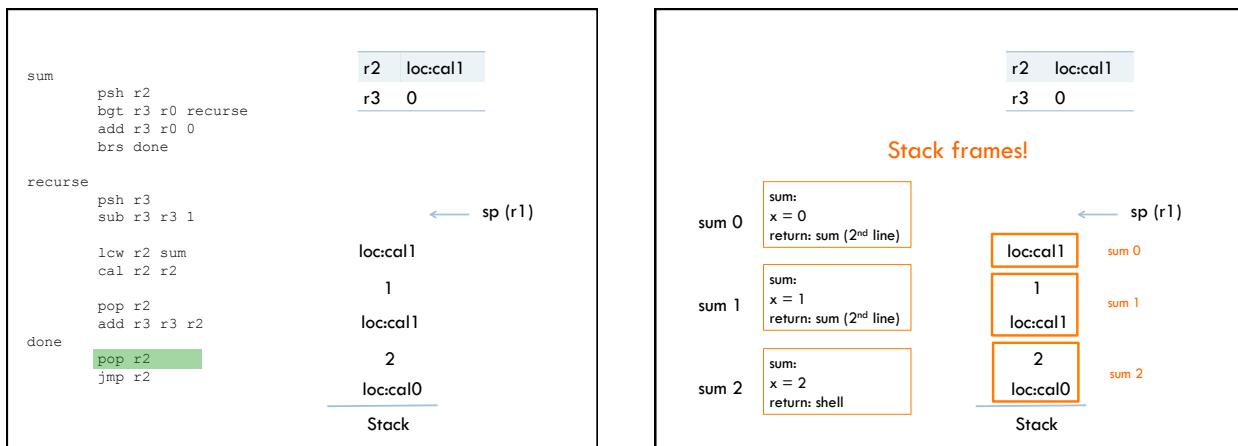
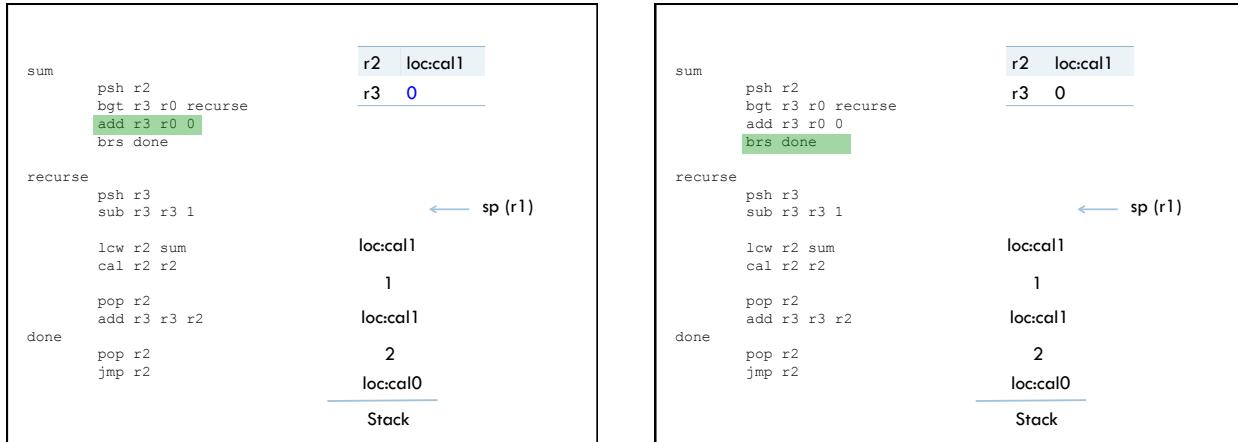


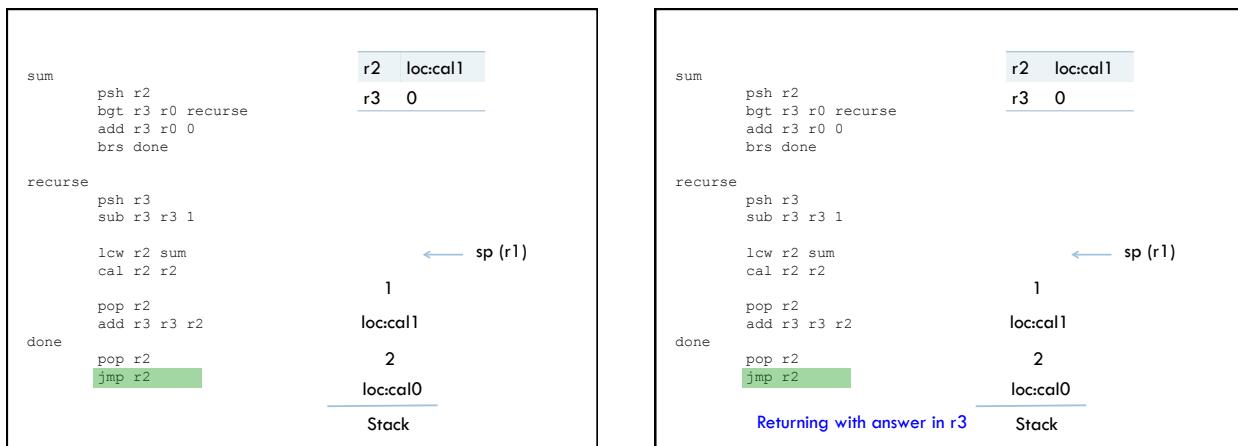
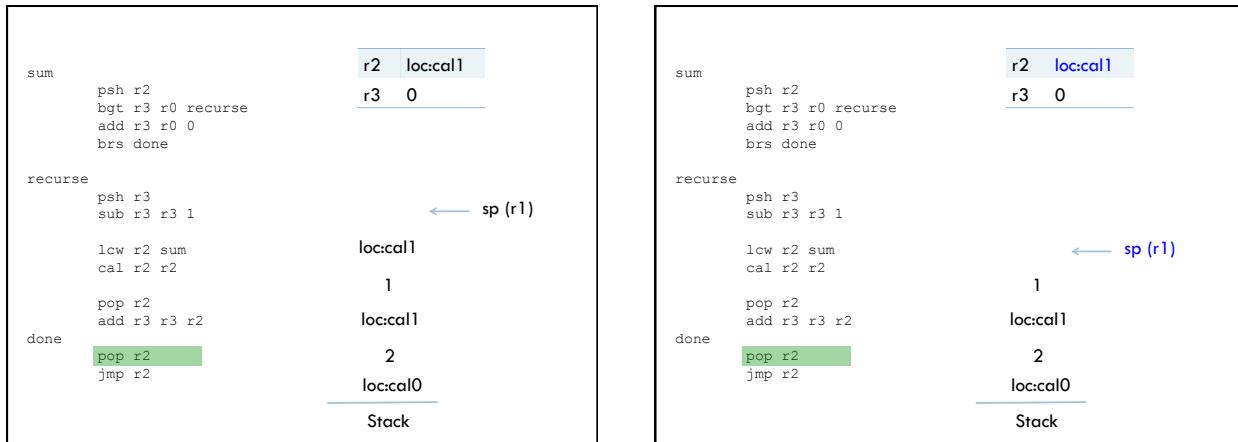


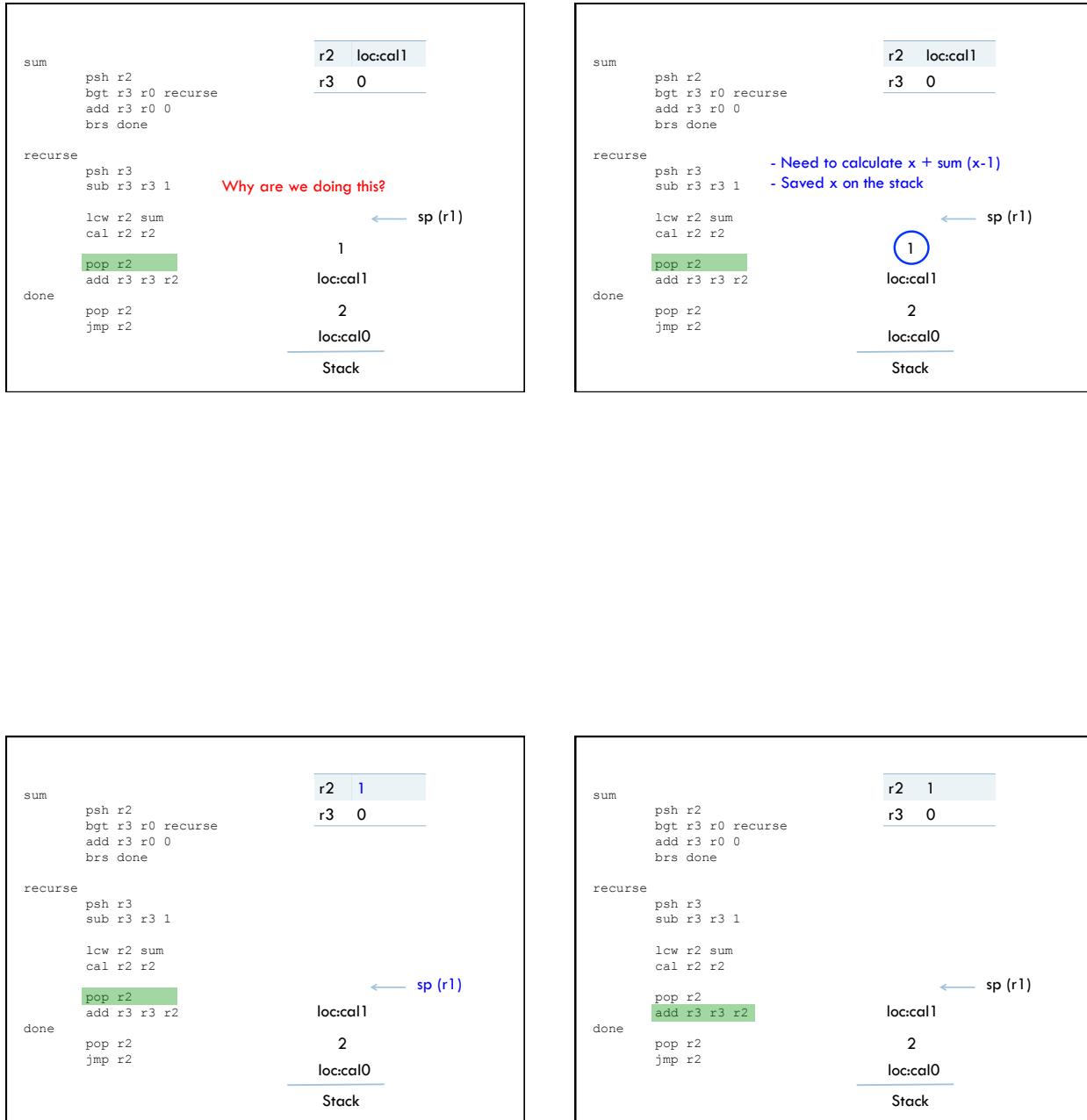


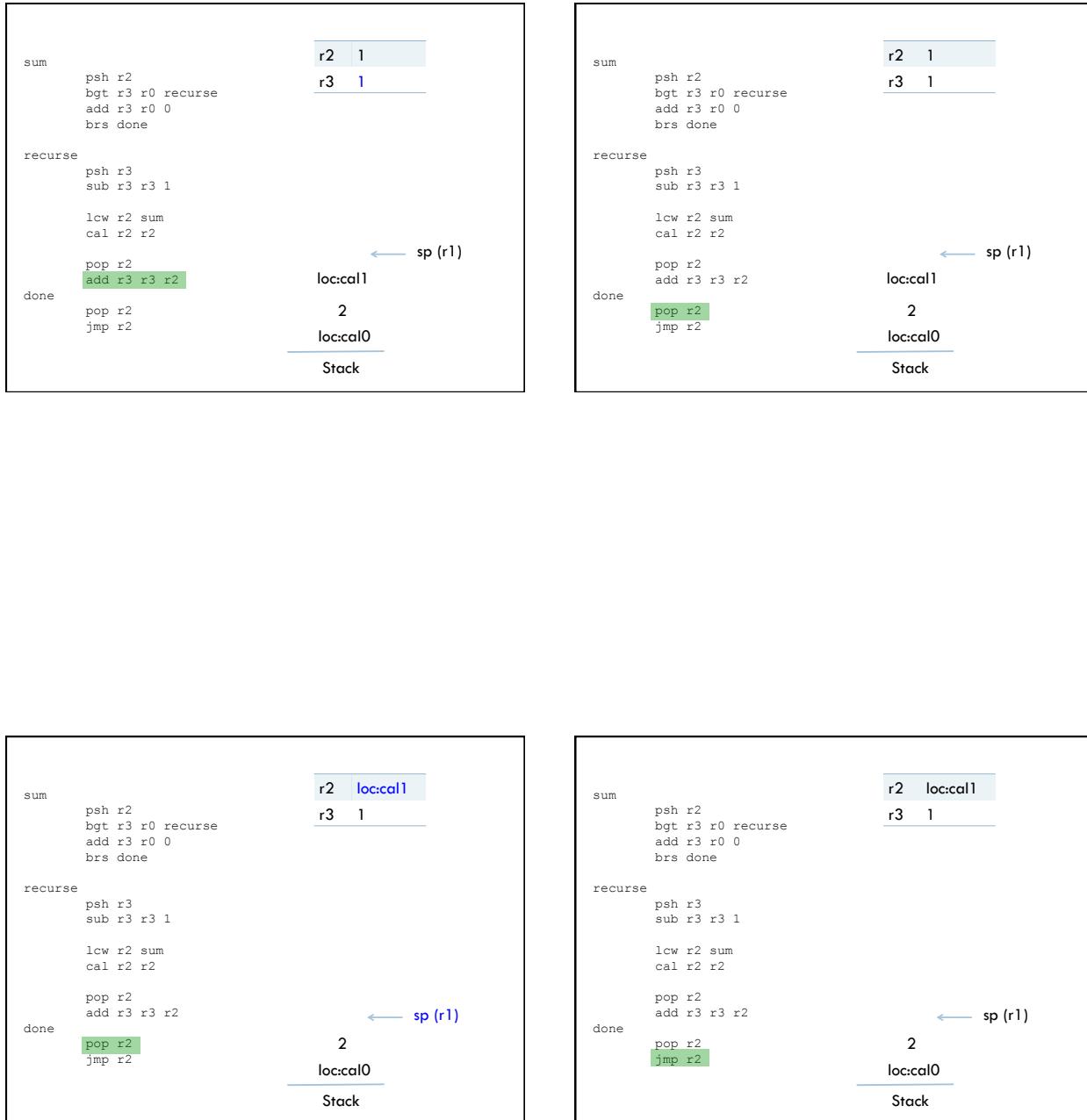


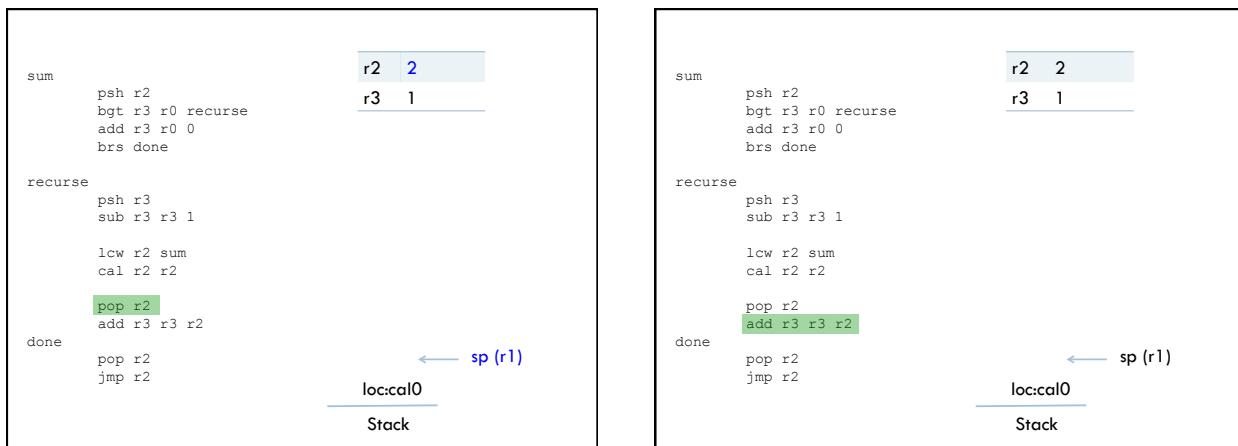
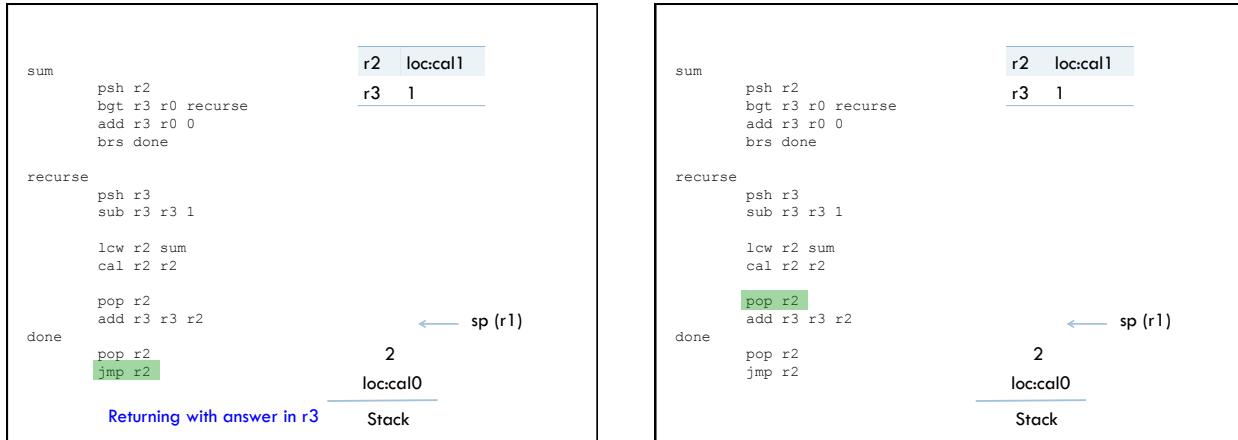




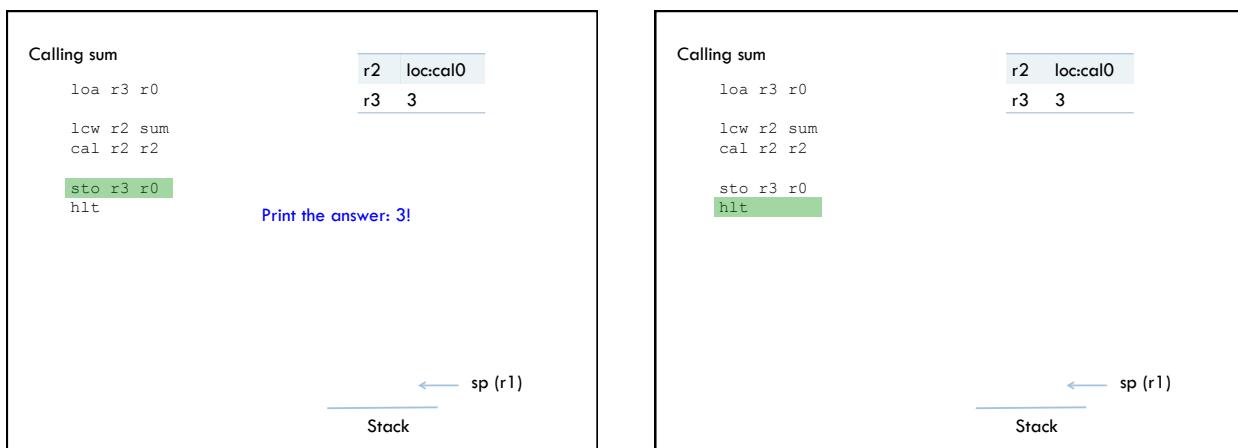
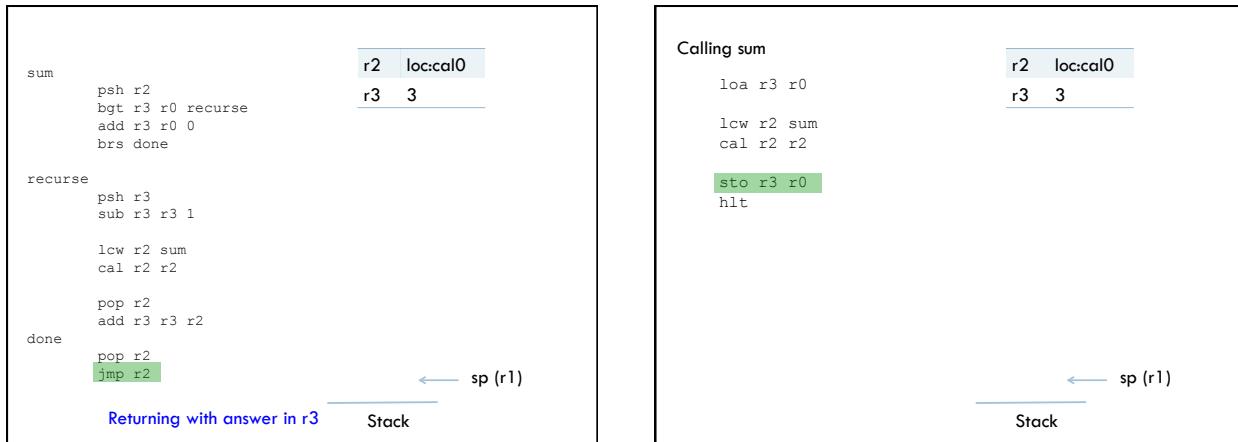












**Calling sum**

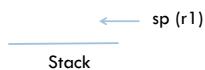
```

loa r3 r0
lcw r2 sum
cal r2 r2
sto r3 r0
hlt

```

r2	loc:cal0
r3	3

Notice that when we're all done, the stack is empty

**Real structure of CS52 program**

```

; great comments at the top!
;
lcw r1 stack           Save address of highest end
                        ; (highest address) of the stack in r1

instruction1      ; comment
instruction2      ; comment
...
;
; stack area: 50 words
;
dat 100
stack

```

} Reserve 50 words for the stack

**Structure of a single parameter function**

```

fname
    psh r2          ; save return address on stack
    ...
    ; do work using r3 as argument
    ; put result in r3
    pop r2          ; restore return address from stack
    jmp r2          ; return to caller

```

**conventions:**

- argument is in r3
- r1 is off-limits since it's used for the stack pointer
- return value goes in r3

**Functions with multiple arguments**

```

fname
    psh r2          ; save return address on stack
    loa r2 r1 4     ; load the second parameter into r2
    ...
    ; do work using r3 and r2 as arguments
    ; put result in r3
    pop r2          ; restore return address from stack
    jmp r2          ; return to caller

```

**conventions:**

- first argument is in r3
- r1 is off-limits since it's used for the stack pointer
- return value goes in r3

## Functions with multiple arguments

```

fframe
psh r2          ; save return address on stack
loa r2 r1 4    ; load the second parameter into r2
...
; do work using r3 and r2 as arguments
; put result in r3
pop r2          ; restore return address from stack
jmp r2          ; return to caller

```

$\text{loa } R_a R_b: \quad R_a = \text{mem}[R_b]$   
 $\text{loa } R_a R_b S: \quad R_a = \text{mem}[R_b + S]$

What does this operation do? What is the 4?

## Functions with multiple arguments

```

fframe
psh r2          ; save return address on stack
loa r2 r1 4    ; load the second parameter into r2
...
; do work using r3 and r2 as arguments
; put result in r3
pop r2          ; restore return address from stack
jmp r2          ; return to caller

```

$\text{loa } R_a R_b: \quad R_a = \text{mem}[R_b]$   
 $\text{loa } R_a R_b S: \quad R_a = \text{mem}[R_b + S]$

- r1 is the stack pointer and points at the top (next) slot
- stacks grow towards smaller memory values

## Functions with multiple arguments

```

fframe
psh r2          ; save return address on stack
loa r2 r1 4    ; load the second parameter into r2
...
; do work using r3 and r2 as arguments
; put result in r3
pop r2          ; restore return address from stack
jmp r2          ; return to caller

```

- r1 is the stack pointer and points at the top (next) slot
- stacks grow towards smaller memory values
- r1+2 is then the top value of the stack
- r1+4 is the 2<sup>nd</sup> value of the stack

## Another recursive example

```

int mystery(int a, int b){
    if( b <= 0 ){
        return 0
    }
    else
        return a + mystery(a, b-1)
}
}

```

What does this function do?

## Recursion

```
int mystery(int a, int b){
    if( b <= 0 ){
        return 0
    } else
        return a + mystery(a, b-1)
}
```

Multiplication...  $a * b$  (assuming b is positive)

Note to future Dave from past Dave: write the function up on the board ☺

```
mult
psh r2      ; save the return address
loa r2 r1 4  ; get at the 2nd argument, b
; a = r3, b = r2
}
Function startup
```

```
bgt r2 r0 else ; r2 > 0, i.e. recursive case
add r3 r0 0   ; return 0
brs endif
}
Base case
```

```
else
sub r2 r2 1  ; r2 = b-1
}
Recursive case
```

```
psh r3      ; save first argument, a, on stack
; (it's going to get overwritten by the return!)
psh r2      ; add r2 as 2nd argument, r3 shouldn't have changed
lcv r2 mult ; call mult recursively
cal r2 r2
pop r0      ; pop 2nd argument off stack
}
answer calculation
```

```
pop r2      ; pop 'a' into r2 off of the stack
add r3 r3 r2 ; r3 = a + mult(a, b-1)
}
Recursive call
```

```
endif
pop r2      ; get the return address
jmp r2      ; return
}
Function cleanup and return
```

```
mult
psh r2      ; save the return address
loa r2 r1 4  ; get at the 2nd argument, b
; a = r3, b = r2
}
Function startup
```

```
bgt r2 r0 else ; r2 > 0, i.e. recursive case
add r3 r0 0   ; return 0
brs endif
}
if( b <= 0 )
return 0
```

```
else
sub r2 r2 1  ; r2 = b-1
}
mystery(a, b-1)
```

```
psh r3      ; save first argument, a, on stack
; (it's going to get overwritten by the return!)
psh r2      ; add r2 as 2nd argument, r3 shouldn't have changed
lcv r2 mult ; call mult recursively
cal r2 r2
pop r0      ; pop 2nd argument off stack
}
a + mystery(a, b-1)
```

```
pop r2      ; pop 'a' into r2 off of the stack
add r3 r3 r2 ; r3 = a + mult(a, b-1)
}
Function cleanup and return
```

```
mult
psh r2      ; save the return address
loa r2 r1 4  ; get at the 2nd argument, b
; a = r3, b = r2
}
Function startup
```

```
bgt r2 r0 else ; r2 > 0, i.e. recursive case
add r3 r0 0   ; return 0
brs endif
}
Notice symmetry of psh and pop
```

```
else
sub r2 r2 1  ; r2 = b-1
}
mystery(a, b-1)
```

```
psh r3      ; save first argument, a, on stack
; (it's going to get overwritten by the return!)
psh r2      ; add r2 as 2nd argument, r3 shouldn't have changed
lcv r2 mult ; call mult recursively
cal r2 r2
pop r0      ; pop 2nd argument off stack
}
a + mystery(a, b-1)
```

```
pop r2      ; load a into r2 off of the stack
add r3 r3 r2 ; r3 = a + mult(a, b-1)
}
answer calculation
```

```
endif
pop r2      ; get the return address
jmp r2      ; return
}
Function cleanup and return
```

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2
r3

← sp (r1)

Stack

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2
r3

← sp (r1)

Stack

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2
r3 6

← sp (r1)

Stack

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2
r3 6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r0
```

```
sto r3 r0
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r0
```

```
sto r3 r0
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r0
```

```
sto r3 r0
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r0
```

```
sto r3 r0
hlt
```

r2	2
r3	6

← sp (r1)

Stack

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2	loc: mult
r3	6

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r0

sto r3 r0
hlt

```

r2	loc: mult
r3	6

**mult**

```

psh r2
loa r2 r1 4

bgt r2 r0 else
add r3 r0 0
brs endif
else
sub r2 r2 1

psh r3
psh r2
lcw r2 mult
cal r2 r2
pop r0

pop r2
add r3 r3 r2
endif
pop r2
jmp r2

```

r2	loc: cal0
r3	6

**mult**

```

psh r2
loa r2 r1 4

bgt r2 r0 else
add r3 r0 0
brs endif
else
sub r2 r2 1

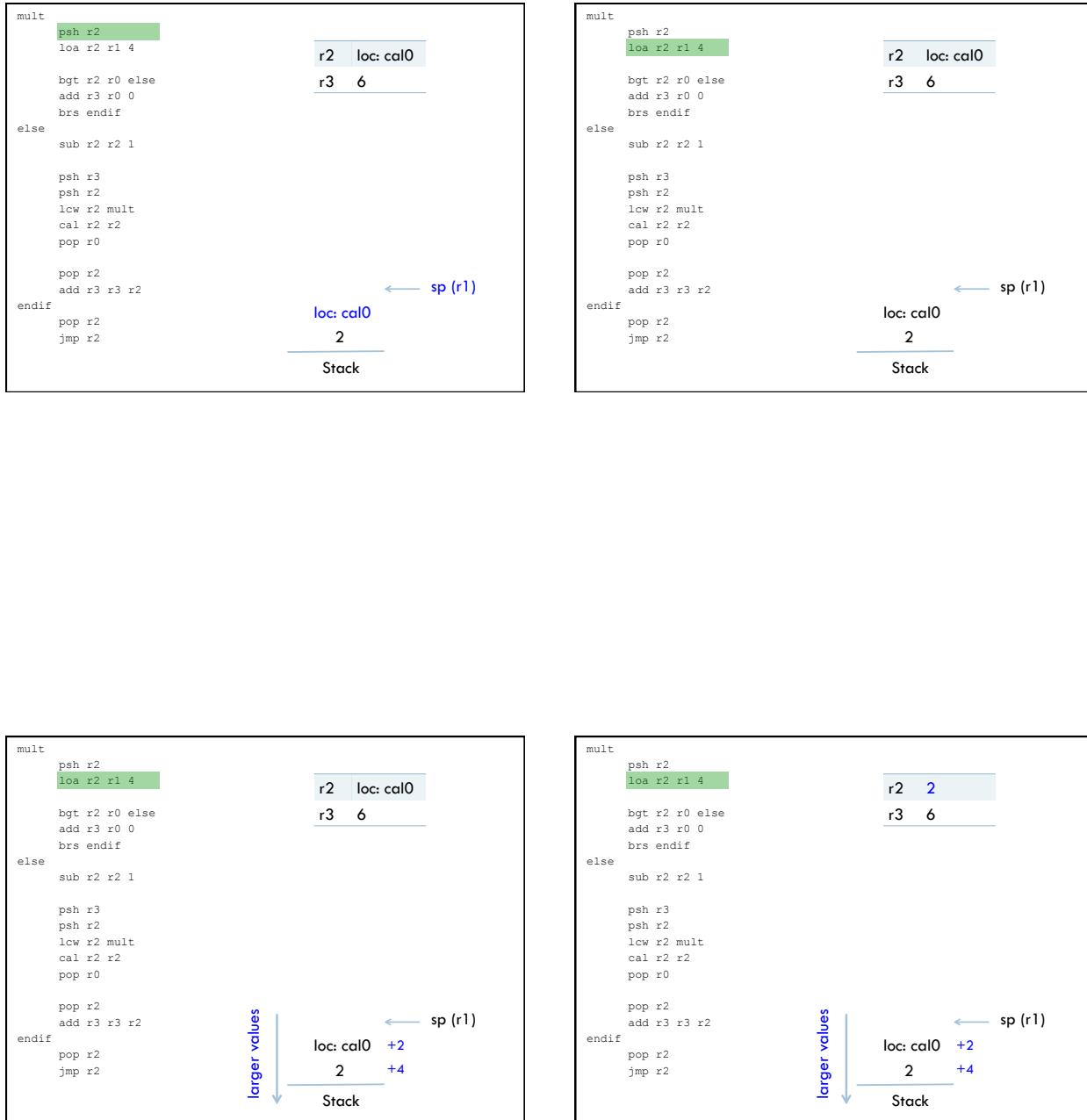
psh r3
psh r2
lcw r2 mult
cal r2 r2
pop r0

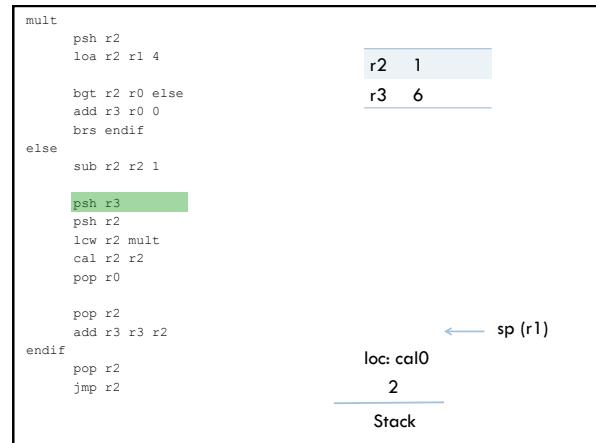
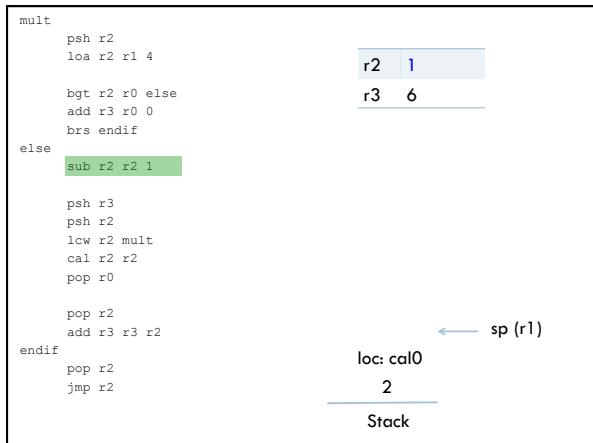
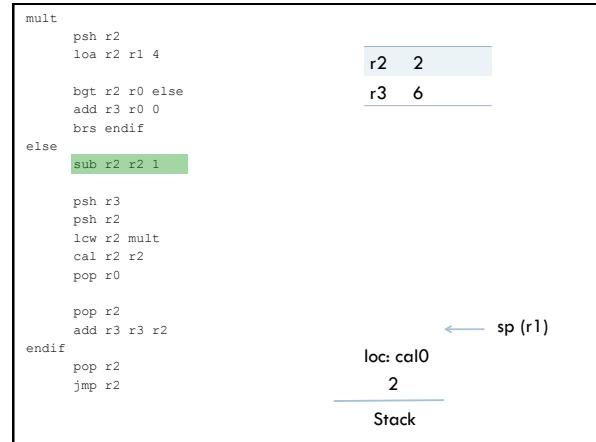
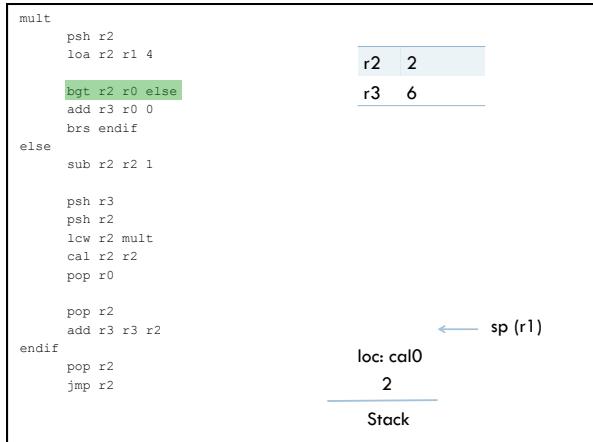
pop r2
add r3 r3 r2
endif
pop r2
jmp r2

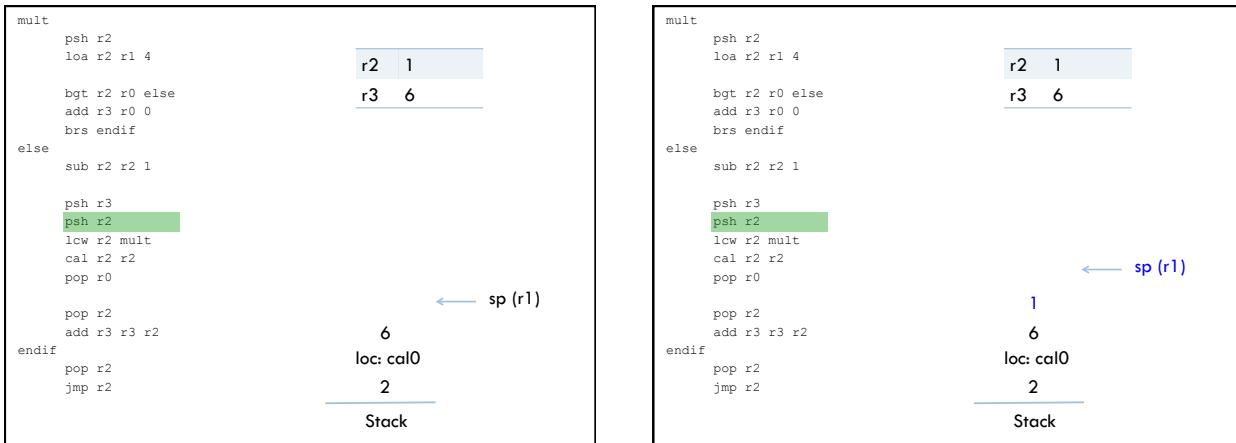
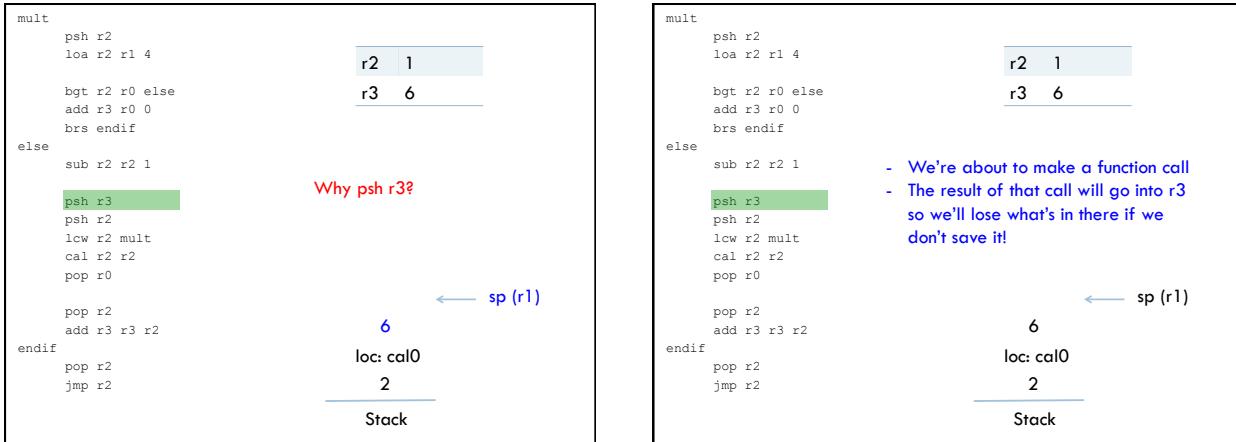
```

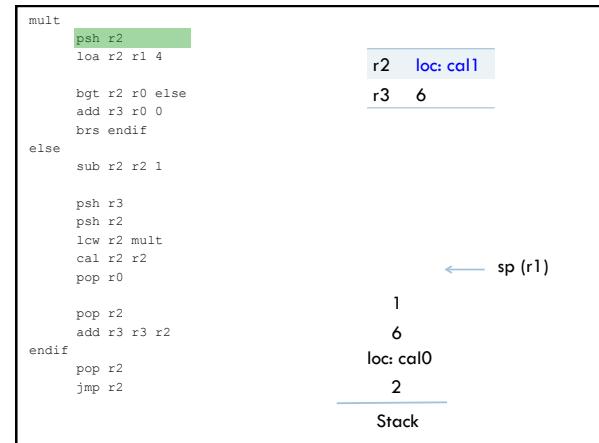
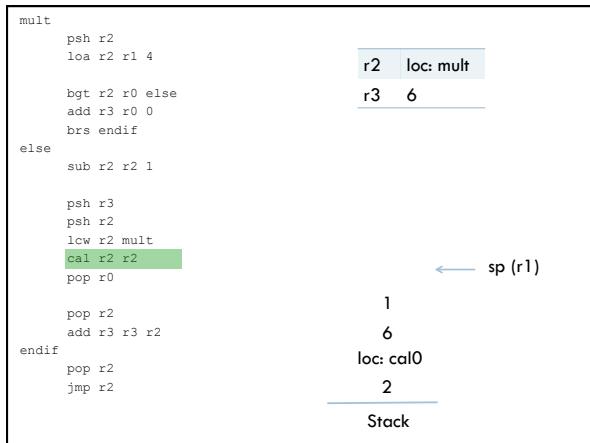
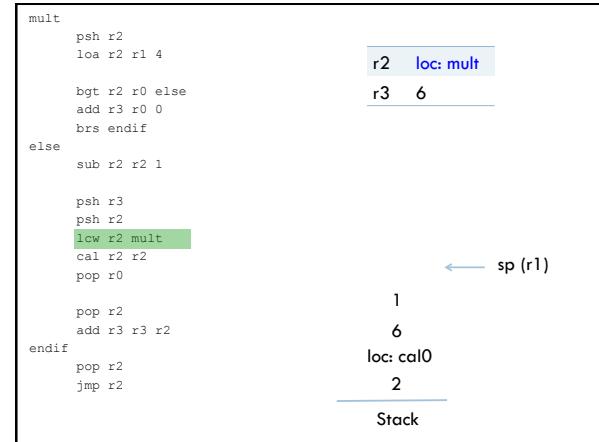
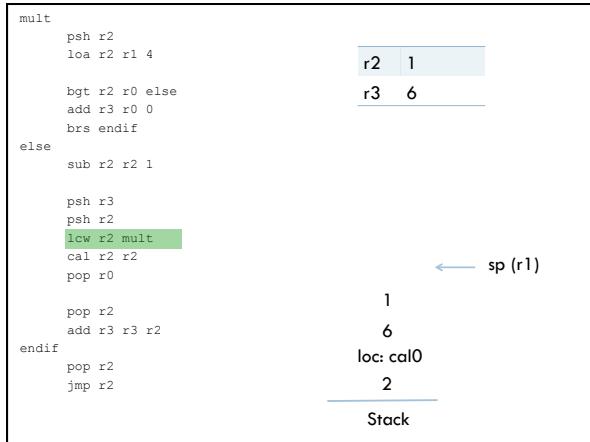
r2	loc: cal0
r3	6

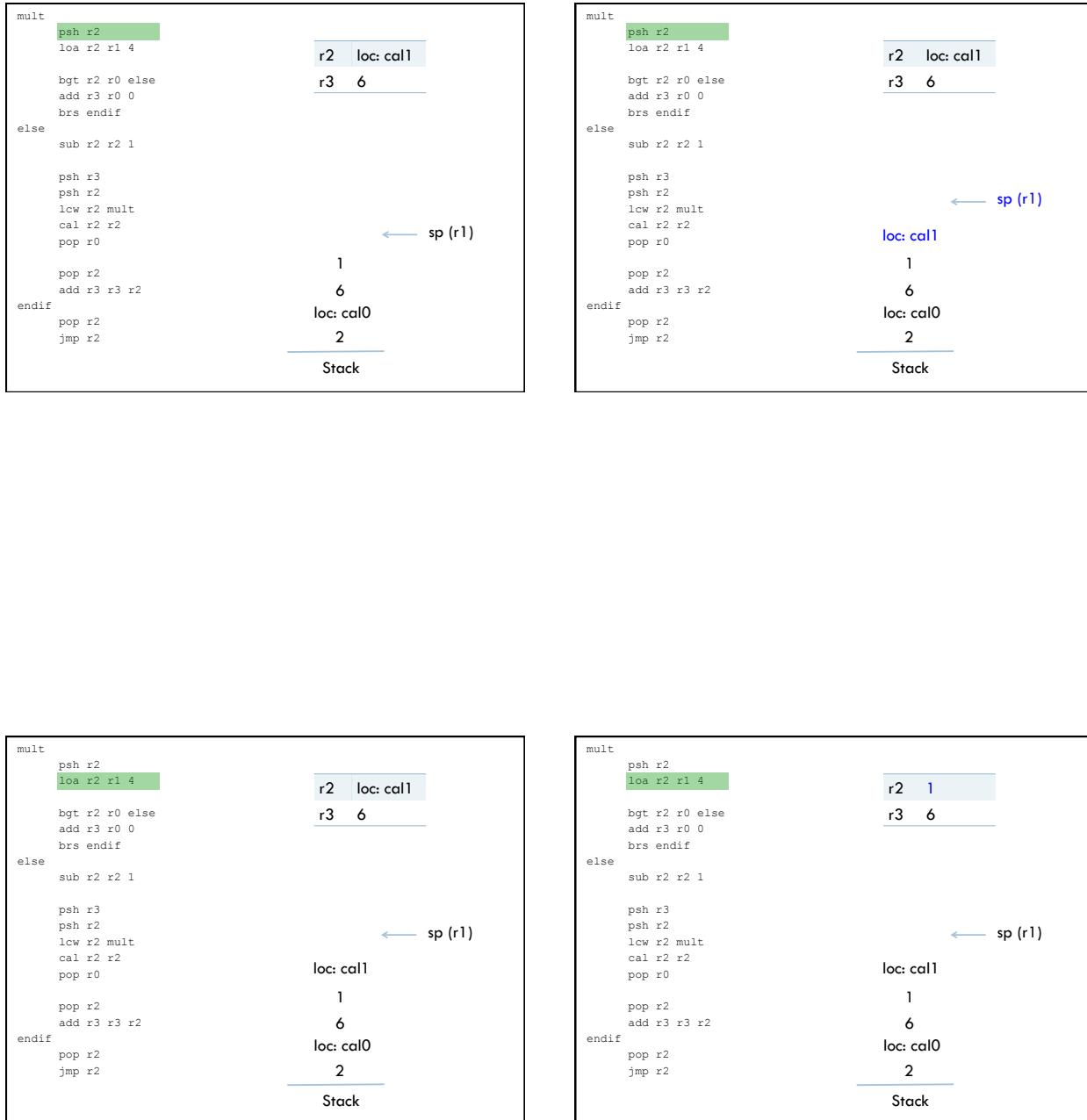


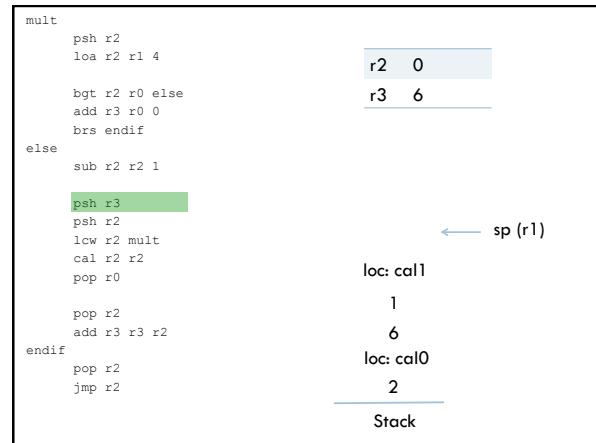
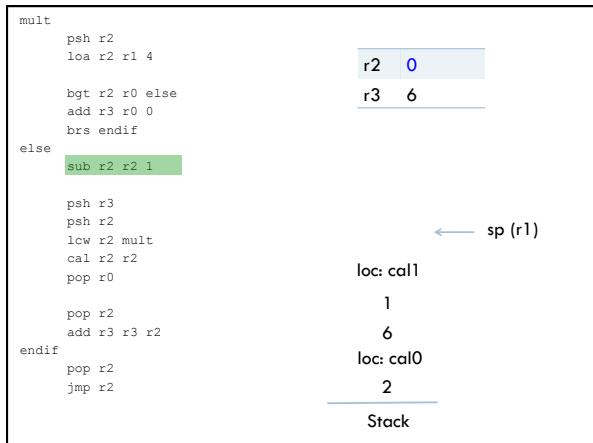
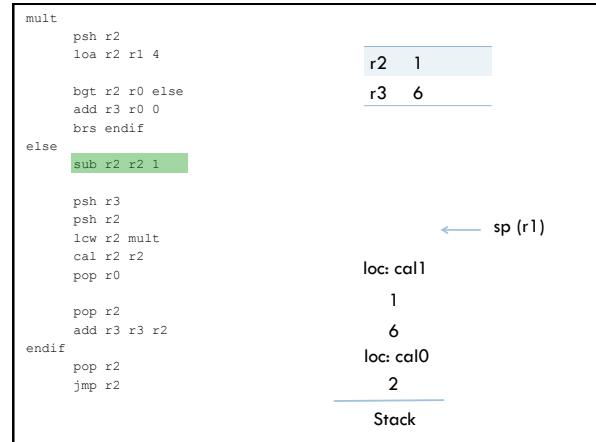
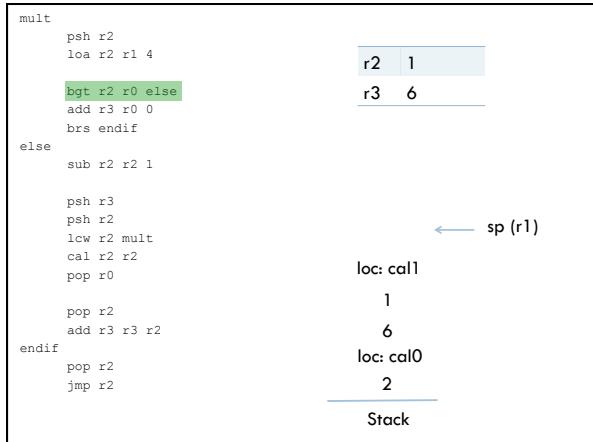


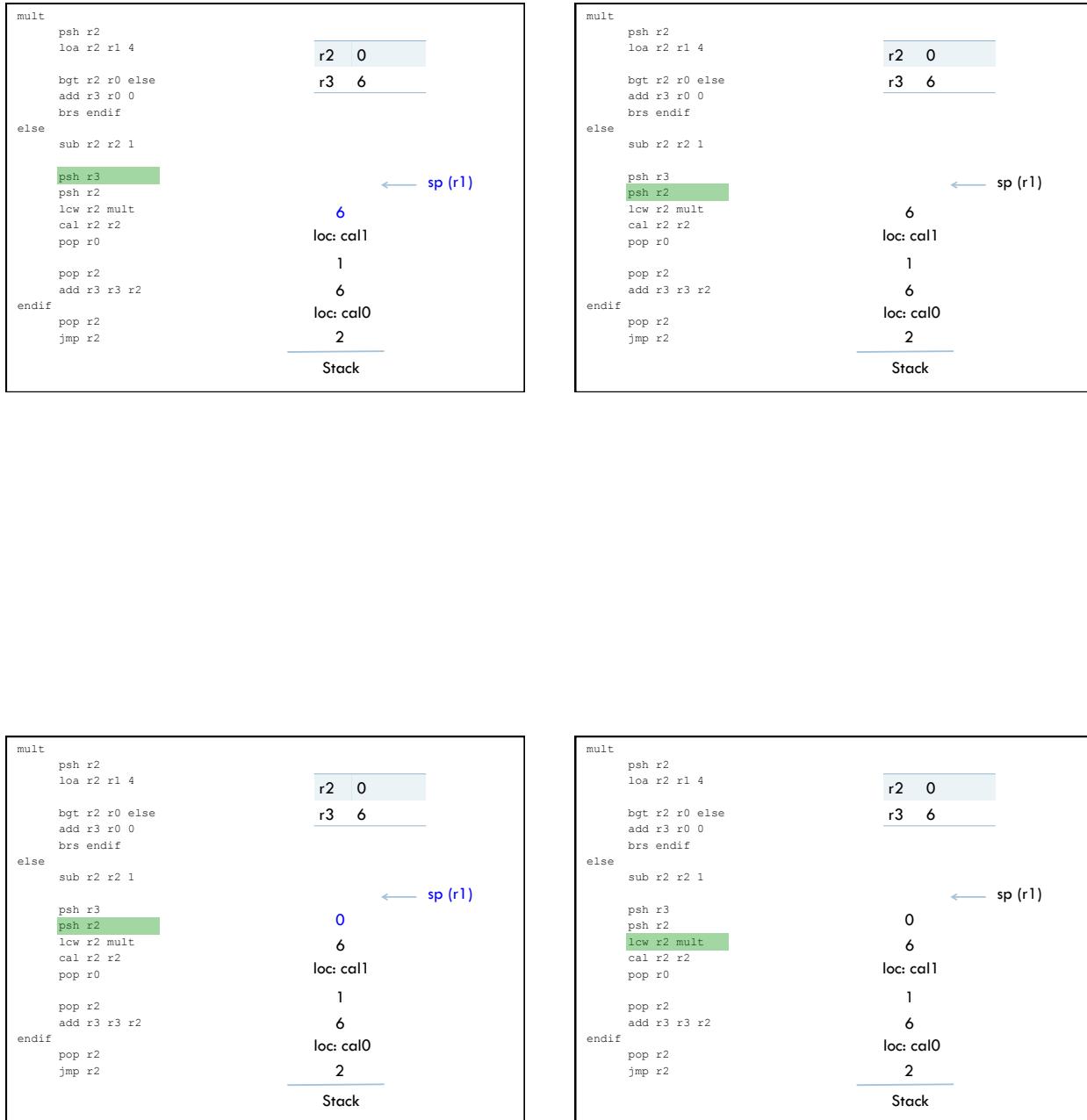


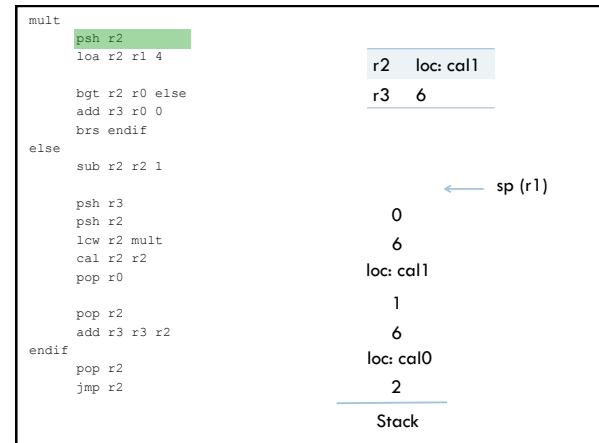
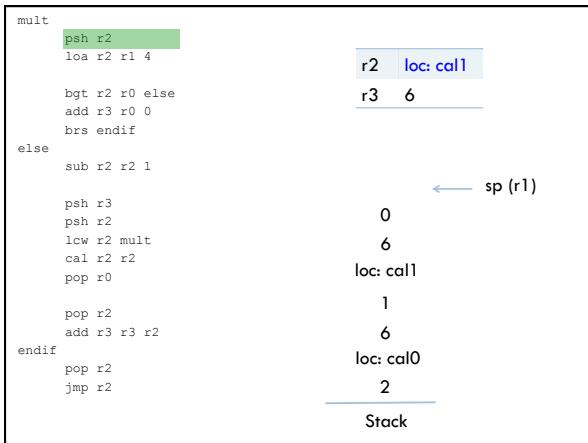
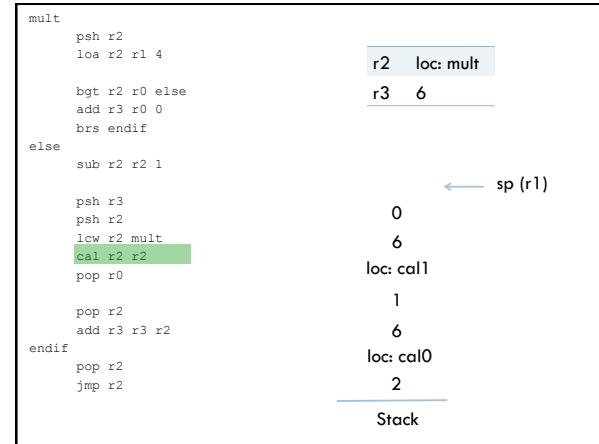
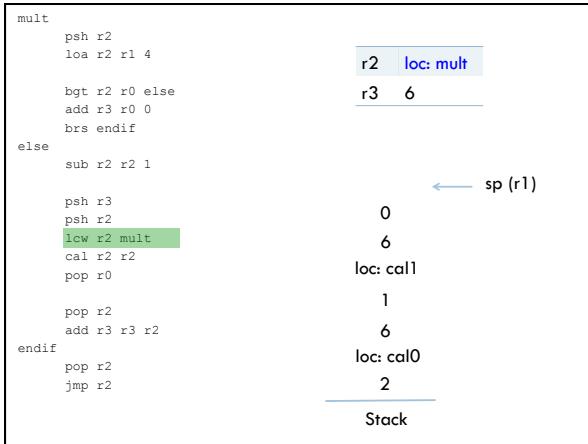


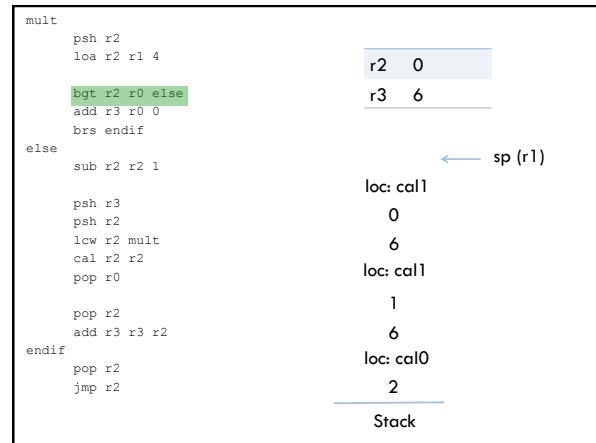
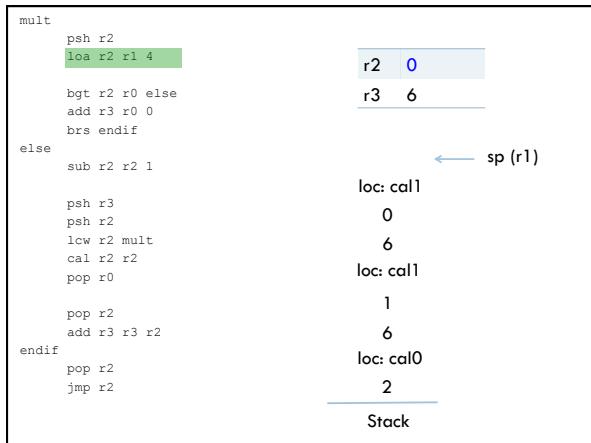
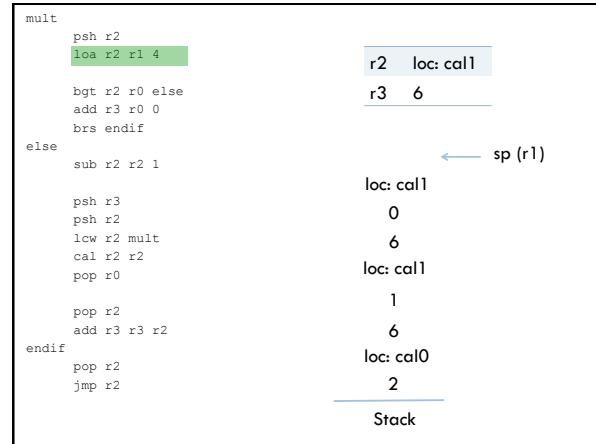
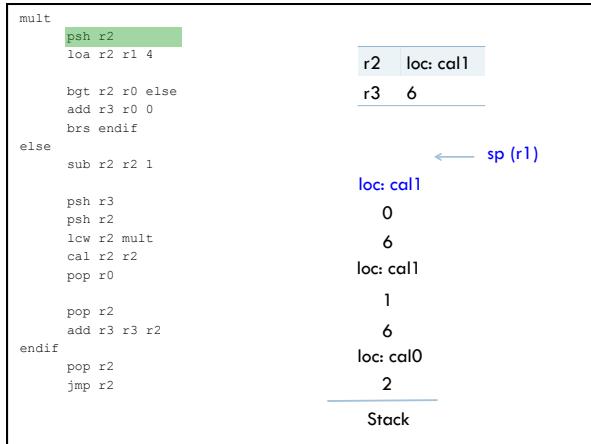


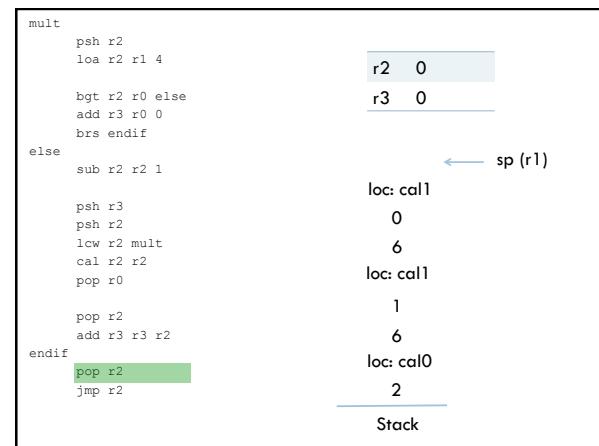
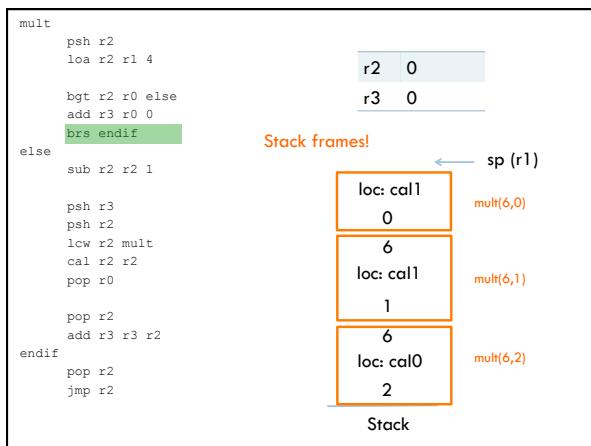
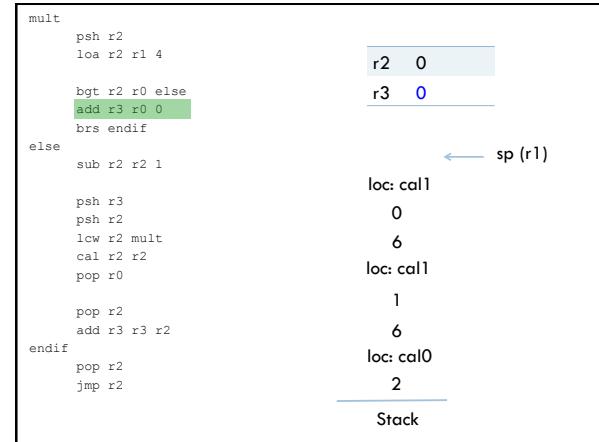
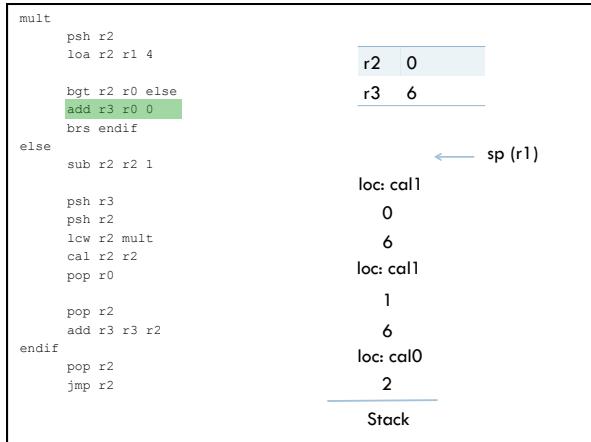


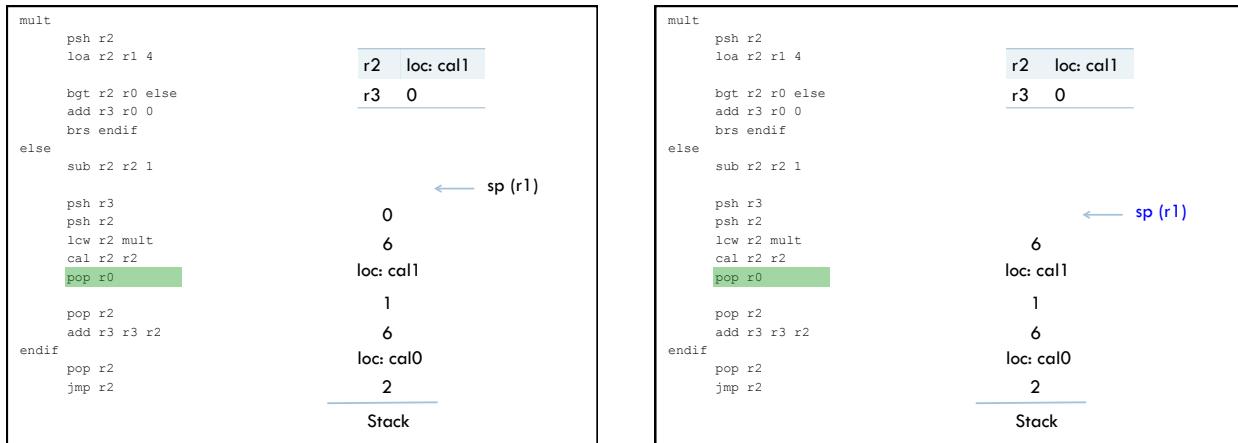
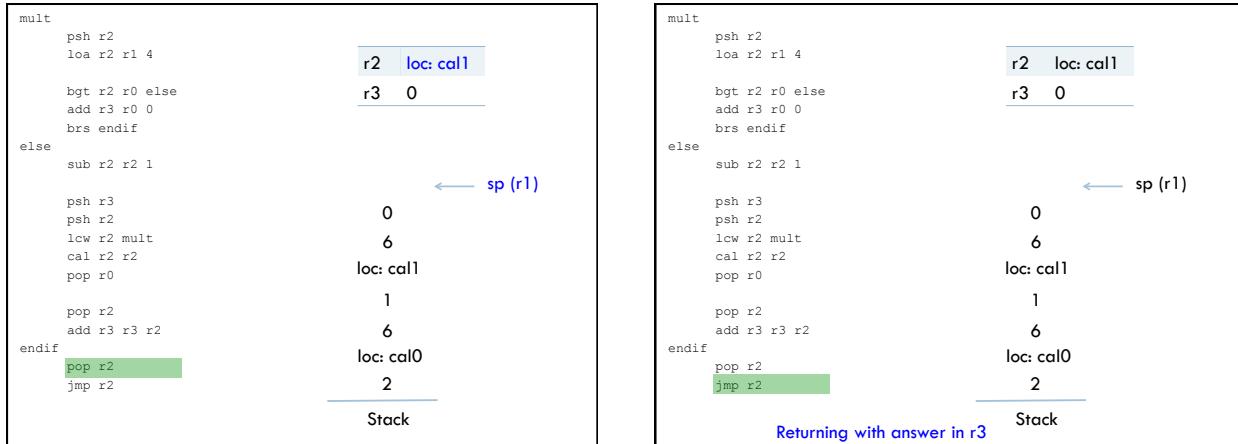


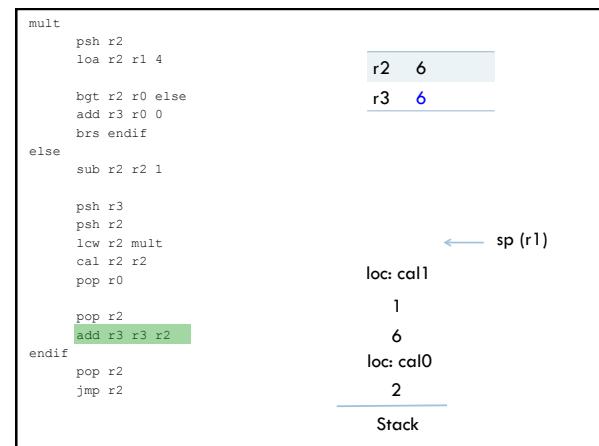
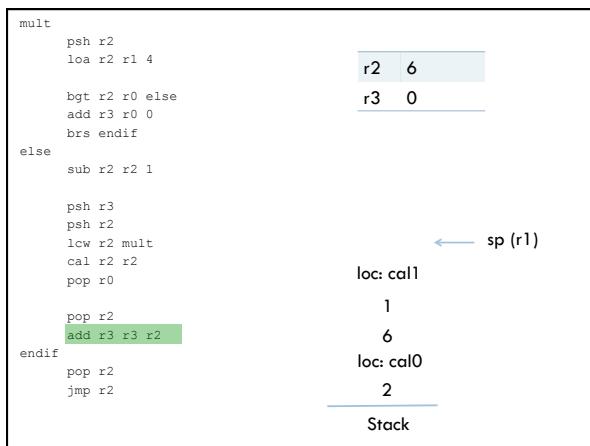
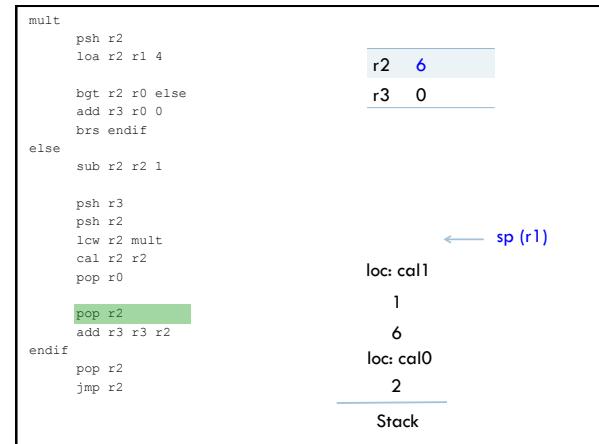
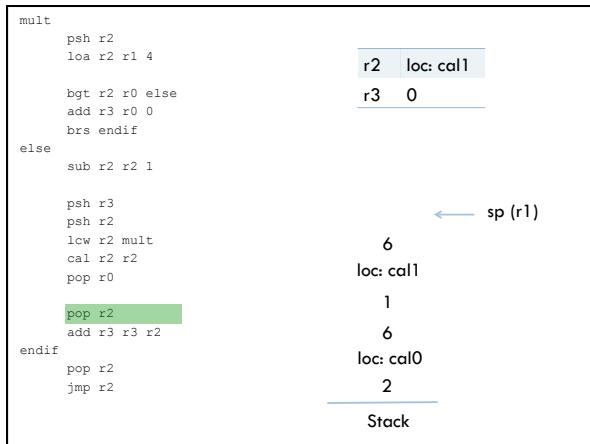


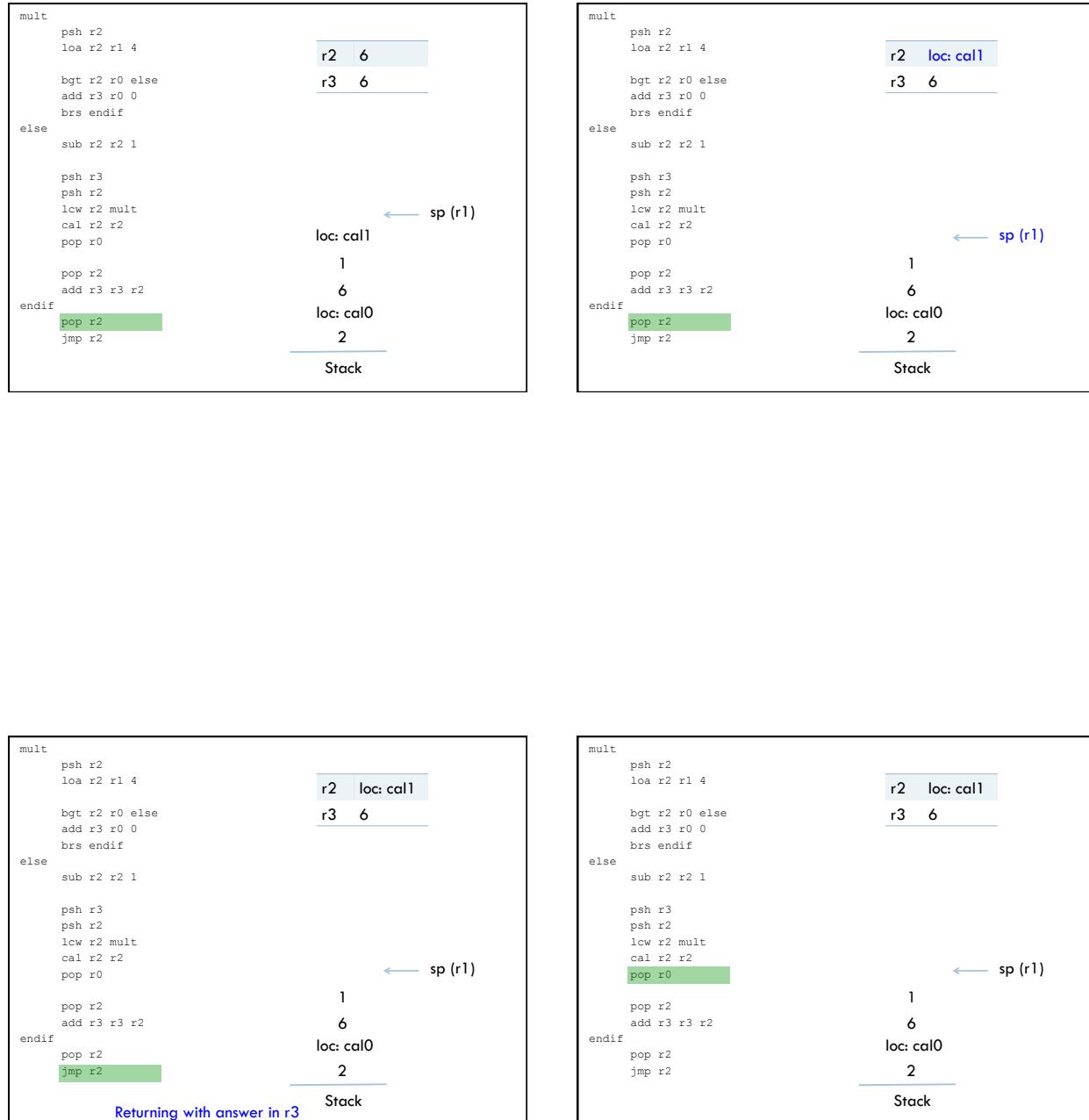


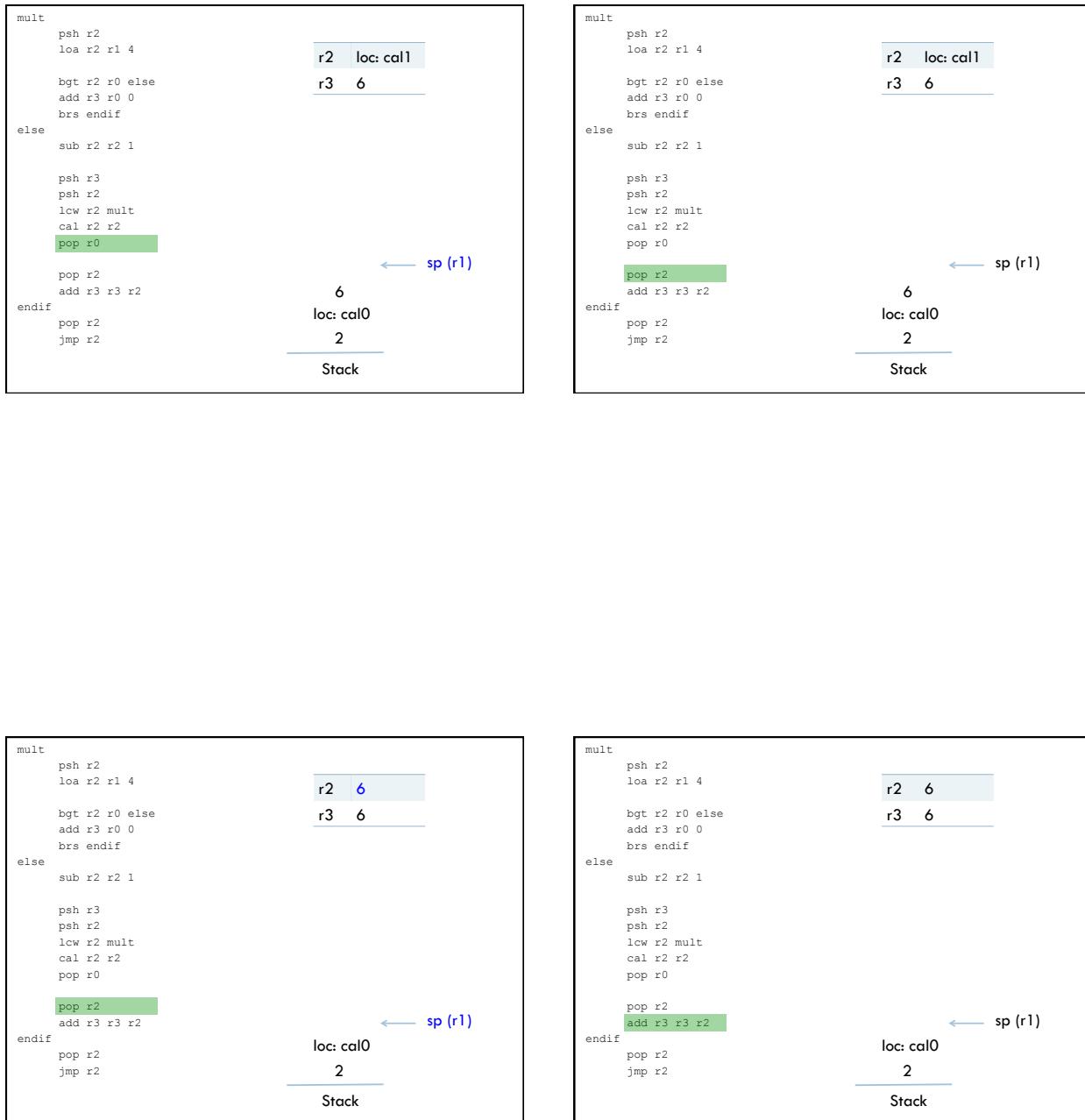


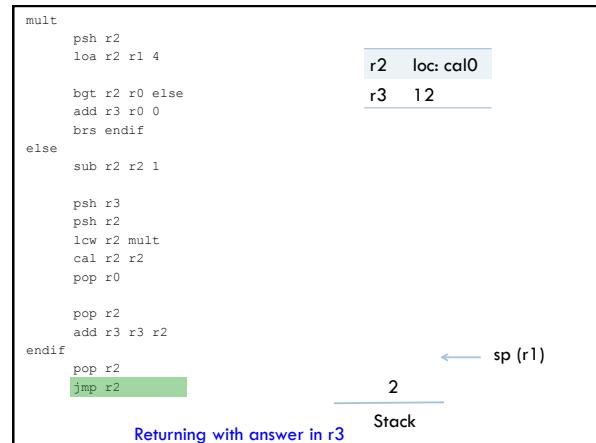
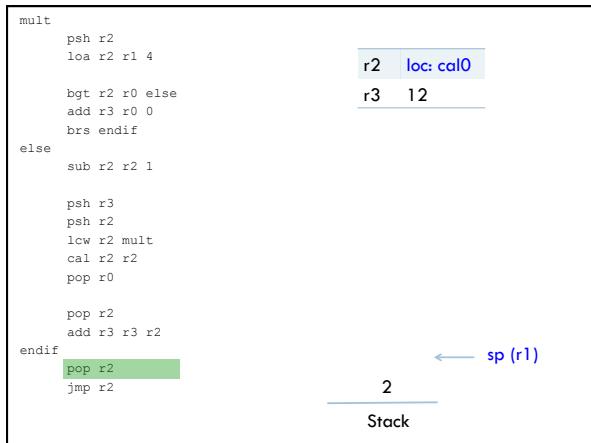
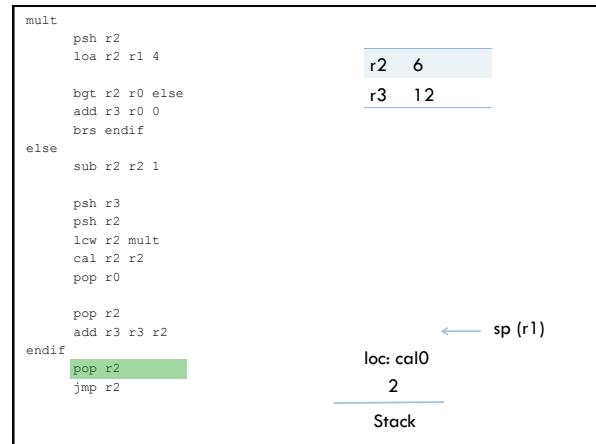
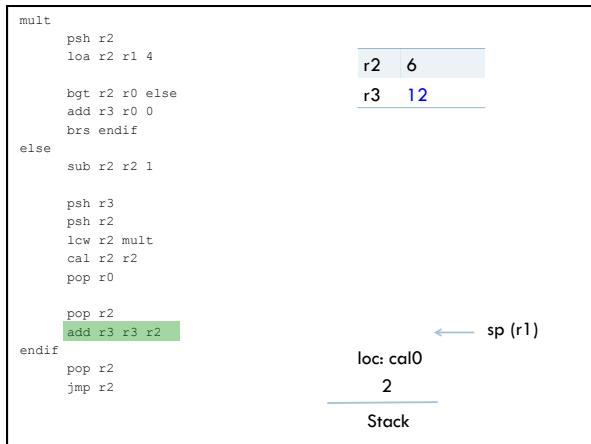


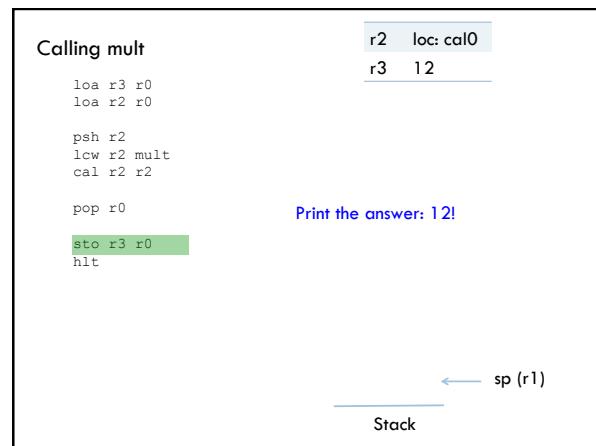
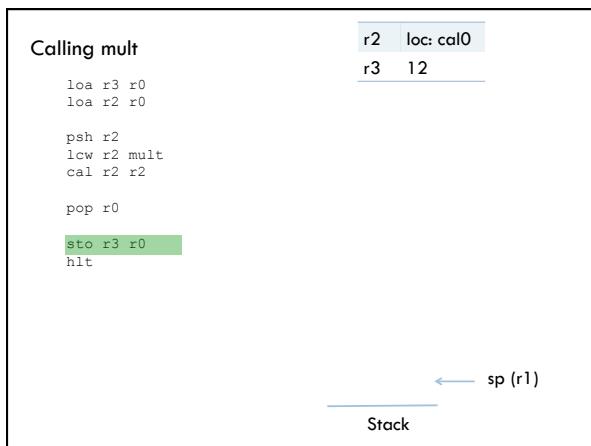
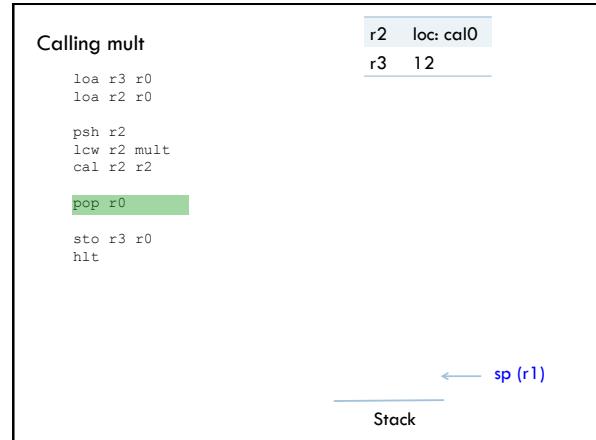
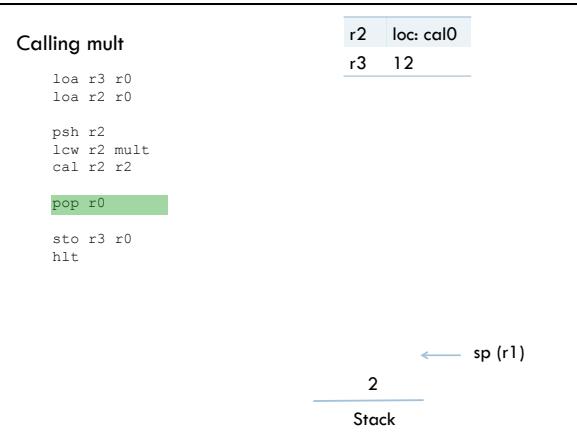












**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lwz r2 mult
cal r2 r2

pop r0
sto r3 r0
hlt

```

r2	loc: cal0
r3	12

Print the answer: 12!

← sp (r1)

Stack

**multiply the easy way**look at `mult_easy.a52` code

we can import libraries (really just functions in other files) using the “inc” command

the included files must be in the same directory as the .a52 file running

**A final aside**

$10101_2$   
 $10_2$   
 $11_2$   
 $1_2$   
 $1010_2$   
 $1010100100100100011111101001_2$

Which of these binary numbers is even?

**A quick aside**

$10101_2 = 21$   
 $10_2 = 2$   
 $11_2 = 3$   
 $1_2 = 1$   
 $1010_2 = 10$   
 $1010100100100100011111101001_2 = 5,675,487,209$

Is there an easier way to tell than just calculating the value?

## A quick aside

The last digit represents the  $2^0 = 1s$  digit

**All other digits represent even values since they are powers of 2**

Therefore:

- If the rightmost digit is 1 = odd number
- If the rightmost digit is 0 = even number

## Bitwise logical operators

instruction name      arguments

add	
sub	
and	RRR or RRS
orr	
xor	

Perform **and**, **or** and **xor** per bit of the number

## Bitwise logical operators

```
add r2 r0 3
add r3 r0 6
and r3 r3 r2
```

## Bitwise logical operators

```
add r2 r0 3
add r3 r0 6
and r3 r3 r2
```

What are r2 and r3 in binary after these instructions?

## Bitwise logical operators

`add r2 r0 3`

`add r3 r0 6`

`and r3 r3 r2`

r2: 11  
r3: 110

## Bitwise logical operators

`add r2 r0 3`

`add r3 r0 6`

`and r3 r3 r2`

Perform **and**, **or** and **xor** per bit of the number

r2: 11  
r3: 110

## Bitwise logical operators

`add r2 r0 3`

`add r3 r0 6`

`and r3 r3 r2`

Perform **and**, **or** and **xor** per bit of the number

r2: 11  
r3: 110

## Bitwise logical operators

`add r2 r0 3`

`add r3 r0 6`

`and r3 r3 r2`

Perform **and**, **or** and **xor** per bit of the number

r2: 11  
r3: 110

0

## Bitwise logical operators

add r2 r0 3

add r3 r0 6

**and r3 r3 r2** Perform **and**, **or** and **xor** per bit of the number

r2:	11
r3:	110
<hr/>	
	0

## Bitwise logical operators

add r2 r0 3

add r3 r0 6

**and r3 r3 r2** Perform **and**, **or** and **xor** per bit of the number

r2:	11
r3:	110
<hr/>	
	10

## Bitwise logical operators

add r2 r0 3

add r3 r0 6

**and r3 r3 r2** Perform **and**, **or** and **xor** per bit of the number

r2:	11
r3:	110
<hr/>	
	10

## Bitwise logical operators

add r2 r0 3

add r3 r0 6

**and r3 r3 r2** Perform **and**, **or** and **xor** per bit of the number

r2:	011
r3:	110
<hr/>	
	10

## Bitwise logical operators

add r2 r0 3

add r3 r0 6

and r3 r3 r2      Perform **and**, **or** and **xor** per bit of the number

r2:	011
r3:	110
<hr/>	
	010

## CS52 programming advice

1. Match your psh and pops
2. Follow the register conventions
3. Develop code incrementally
4. Debugging: write out stack, registers, etc. on paper and compare against system execution

## Examples from this lecture

<http://www.cs.pomona.edu/~dkouchak/classes/cs52/examples/cs41b/>

## Another example

I didn't have time to cover the next example in class, but left it in the notes as another example of a function that takes two parameters

### Another example

```

max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif      What does this code do?
    add r3 r2 0
endif
    pop r2
    jmp r2

```

### Another example

```

max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif      max, as a function!
    add r3 r2 0
endif
    pop r2
    jmp r2

```

### Calling max

```

loa r3 r0
loa r2 r0
    Anything different?
psh r2
lcw r2 max
cal r2 r2
pop r0

sto r3 r0
hlt

```

### Calling max

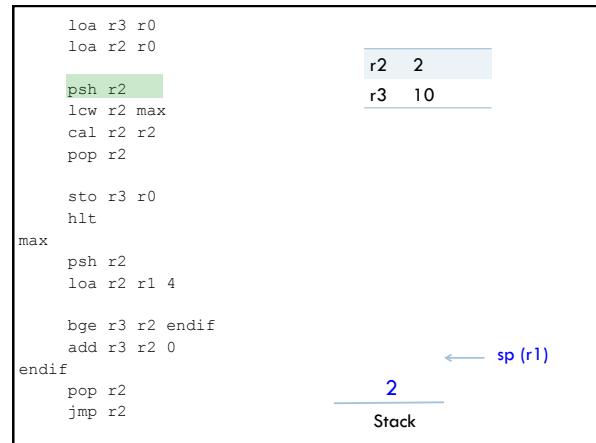
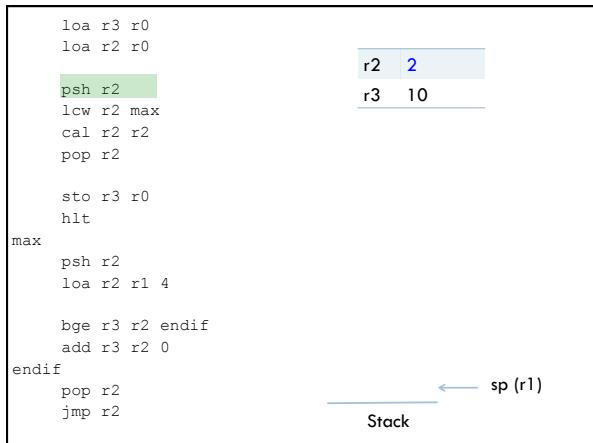
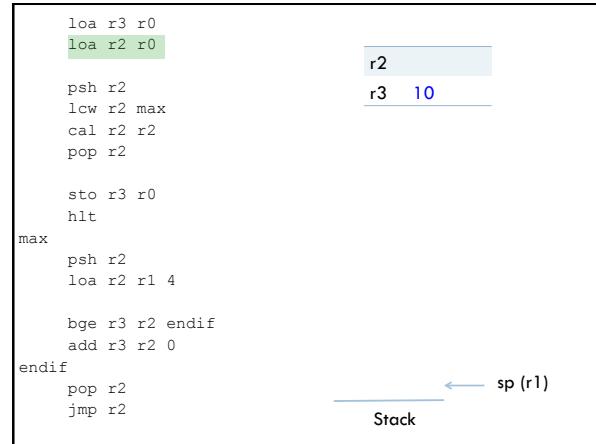
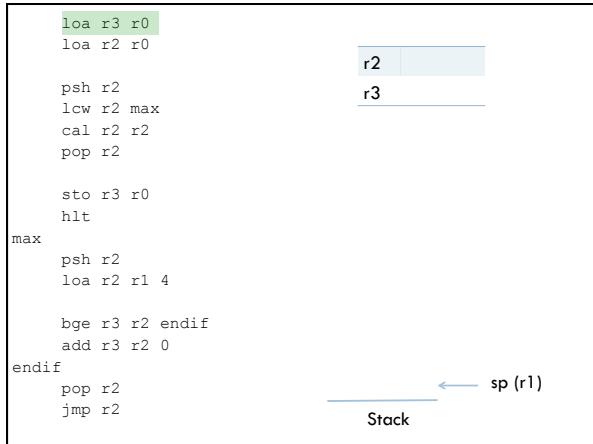
```

loa r3 r0
loa r2 r0

psh r2
lcw r2 max      For the second argument,
cal r2 r2          psh it on the stack
pop r0

sto r3 r0
hlt

```



```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r3 r0
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    add r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2  
Stack

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r3 r0
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    add r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2  
Stack

Notice that we overwrote the value in r2

If we hadn't saved it on the stack, it would have been lost

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r3 r0
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    add r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2  
Stack

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r3 r0
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    add r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2  
Stack

r2 loc: cal

